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Global effective lower bound and unconventional monetary policy☆

Jing Cynthia Wu [a], Ji Zhang [b,*]

[a] University of Notre Dame and NBER, United States of America
[b] PBC School of Finance, Tsinghua University, China

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ABSTRACT

In a standard open-economy New Keynesian model, the effective lower bound causes anomalies: output and terms of trade respond to a supply shock in the opposite direction compared to normal times. We introduce a tractable framework to accommodate for unconventional monetary policy. In our model, these anomalies disappear. We allow unconventional policy to be partially active and asymmetric between countries. Empirically, we find the US, Euro area, and UK have implemented a considerable amount of unconventional monetary policy: the US follows the historical Taylor rule, whereas the others have done less compared to normal times.

1. Introduction

Since the Great Recession, many major central banks of developed economies have faced the effective lower bound (ELB) for their policy interest rates and resorted to unconventional monetary policy to provide further stimulus. In this extraordinary environment, how do we evaluate the role of unconventional monetary policy theoretically and empirically?

In a standard New Keynesian model (e.g., Eggertsson and Woodford (2003) for a closed economy and Cook and Devereux (2013a) for an open economy), the ELB yields to the classic liquidity trap. The central bank cannot further reduce the policy rate, and monetary policy is completely absent. However, emerging empirical studies provide overwhelming evidence to demonstrate the effectiveness of unconventional monetary policy; see, for example, Gagnon et al. (2011), Hamilton and Wu (2012), Krishnamurthy and Vissing-Jorgensen (2011), Bauer and Rudebusch (2014), and Wu and Xia (2016) for its domestic impact, and Neely (2015), Bauer and Neely (2014), Bowman et al. (2015), and Chen et al. (2016) for its global effects.

We propose a tractable New Keynesian model that incorporates unconventional monetary policy into an otherwise standard model to be consistent with empirical findings. We propose a Taylor (1993)-type policy rule to conveniently summarize both conventional and unconventional monetary policy; see Wu and Zhang (2017) for how to implement a negative interest rate via QE, for example. We extend the framework of Wu and Zhang (2017), where unconventional monetary policy follows the historical Taylor rule by construction. In this paper, we relax this assumption and allow unconventional policy to be potentially less effective, and countries can implement them asymmetrically. Our new model nests the traditional model where monetary policy is absent at the ELB and the model in Wu and Zhang (2017) with fully active unconventional monetary policy. We illustrate our new framework with a two-country setup, similar to Clarida et al. (2002) and Cook and Devereux (2013a), but it can be easily extended to the small-open economy.

During normal times, a negative supply shock from the home country leads to lower home output and terms of trade. In our model, if a sufficient amount of unconventional monetary policy is implemented, the same results apply for the ELB. On the contrary, the standard model implies an opposite movement of output and terms of trade during a liquidity trap, and we will refer to these movements as anomalies.

The basic mechanism that leads to these anomalies consists of two channels. First, it transmits through inflation and the real interest rate, which works the same way as in a closed-economy macro model. A negative supply shock leads to higher inflation for home goods. At the ELB, the nominal rate does not move, which lowers the real rate. The lower

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* Corresponding author.
E-mail addresses: cynthia.wu@nd.edu (J.C. Wu), zhangji@pbcsf.tsinghua.edu.cn (J. Zhang).

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real rate stimulates demand and hence the equilibrium output of the home country. In the open economy with complete financial markets, international trade further amplifies this effect through a depreciation in terms of trade.

When we allow the two countries to implement their respective unconventional monetary policy asymmetrically, we find different results for the home and foreign economies. For the home country, its own policy matters the most, whereas the foreign economy relies on both central banks. More active home or foreign policy is associated with higher welfare, and the most efficient case is obtained when both countries' unconventional policies follow their historical policy rules.

We explore alternative model and parameter specifications for robustness. The anomalies are generally robust for alternative models with one exception: the anomaly for terms of trade depends on whether the international financial markets are complete or not, whereas the result for output is not sensitive. We also assess the robustness of these anomalies across alternative parameter values. We find they are not sensitive to structural parameters, including the Frisch elasticity of labor supply, elasticity of intertemporal substitution, and home bias. Results vary more over parameters governing the preference shock, which creates the ELB environment. We find as long as the ELB lasts for several quarters or longer, the anomalies hold.

Finally, we seek empirical evidence for unconventional monetary policy in the United States, Euro area, and United Kingdom. First, we test model implications by comparing how output responds to a supply shock in a structural vector autoregression (VAR) between normal times and the ELB. We find that for all three countries and regions, output decreases with a negative shock to the growth rate of total-factor productivity (TFP) regardless of normal times or the ELB. This result is in contrast to the anomaly presented in the standard New Keynesian model. Our theoretical model suggests unconventional monetary policy as one potential explanation for this result.[1]

Next, we quantify unconventional monetary policy empirically. Specifically, we compare what has been done with what should have been done according to the historical Taylor rule. We find the US, Euro area, and UK have implemented a considerable amount of unconventional monetary policy, which explains why the anomaly does not appear in the data. Moreover, the US operates its unconventional monetary policy similarly to the historical Taylor rule, whereas the Euro area and UK have operated less unconventionally than what they would normally have done.

The rest of the paper after a brief literature review proceeds as follows. Section 2 describes the theoretical model, and we discuss model implications with and without unconventional monetary policy in Section 3. Section 4 assesses empirical evidence for unconventional monetary policy, and Section 5 concludes.

1.1. Literature

Our paper is related to several recent papers that investigate policy responses in the global low interest rate environment. Cook and Devereux (2013a) analyze the interaction between monetary and fiscal policy in a global liquidity trap with a two-country New Keynesian model. Cook and Devereux (2013b) compare a currency union with a system with a flexible exchange rate. Fujiwara et al. (2013) focus on the optimal monetary policy. Eggertsson et al. (2016) consider the effectiveness of monetary and fiscal policy during the global secular stagnation, using an open-economy overlapping generations model. Different from the existing literature, our model focuses on the role of unconventional monetary policy.

Our empirical analysis of the Taylor rule is related to Hakkio and Kahn (2014). The main difference is we propose alternative ways to compute the quantity for what should have been done, and our methods are less subject to accumulating and compounding errors and uncertainty over time. Our structural VAR results are consistent with Garín et al. (forthcoming) and Debortoli et al. (2016). The difference is the literature has focused on the US, and our analysis encompasses the US, Euro area, and UK.

The literature has discussed alternative solutions for the ELB. For example, Boneva et al. (2016) and Gust et al. (2017) propose using nonlinear methods, and Cochrane (2017) recommends exploring alternative equilibria. We focus on unconventional monetary policy as a plausible explanation, and a similar argument has been made to explain ELB with different models. For example, Diba and Loisel (2017) model a liquidity premium as an instrument for unconventional monetary policy. The benefit of our framework is its tractability: It is a straightforward extension of the standard New Keynesian model, and can nest various models proposed in the literature; see Wu and Zhang (2017) for details.

The ELB environment is analogous to a currency union for country-specific shocks; see Eggertsson et al. (2014) and Galí and Monacelli (2016). For a discussion of their slight differences, see Erceg and Lindé (2012). Future work could explore how to carry our framework into that environment.

2. Model

This section describes a two-country open-economy New Keynesian model. Many model ingredients are standard and similar to Clarida et al. (2002) and Cook and Devereux (2013a). The main difference is we do not restrict our attention to the standard setup for the ELB, that is, the nominal interest rate is zero and the monetary authority provides no additional stimulus. Instead, we allow a potential role for unconventional monetary policy, which could be completely inactive, fully active, and anywhere in between. See the setup in Subsection 2.5, and economic implications are discussed in Section 3.

The two countries, home and foreign, are the same size and symmetric. Households consume both home and foreign goods with some preference for the domestically produced products. Firms hire labor to produce differentiated goods, and face Calvo (1983)-type price rigidity. The wage paid to workers is determined in a perfectly competitive labor market without any frictions. Complete financial markets allow perfect international risk sharing.[2] Monetary policy follows a Taylor (1993) rule.

For the most part, we describe the home economy, and the foreign optimization problems are symmetric, which are denoted by an asterisk superscript *. H stands for home-produced goods, and F is foreign goods.

2.1. Households

2.1.1. Optimization problem

Households maximize their utility over consumption and hours worked:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Xi_t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right], \tag{2.1}$$

where $\mathbb{E}$ is the expectation operator and β is the discount factor. Ξ_t is the preference shifter with steady-state value 1, and $\xi_t = \log(\Xi_t)$ follows $\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_{\xi t}$, $\varepsilon_{\xi t} \sim N(0, \sigma_\xi^2)$. C_t is consumption and N_t is labor supply. σ is the elasticity of intertemporal substitution and ϕ is the inverse of the Frisch elasticity of labor supply.

Their budget constraint is

$$P_t C_t + \mathbb{E}_t \left[Q_{t,t+1} B_{t+1} \right] = B_t + W_t N_t, \tag{2.2}$$

where P_t is the consumer price index (CPI) and W_t is nominal wage. B_{t+1} is the period $t+1$ random payoff of the asset bought at t, and $Q_{t,\,t+1}$ is the corresponding stochastic discount factor between t and $t+1$.

[1] Debortoli et al. (2016) use a similar VAR result in the US to argue for unconventional monetary policy.

[2] We will discuss incomplete markets in Section 3.3.4.

Households' Euler equation is

$$\beta\mathbb{E}_t\left[\frac{\Xi_{t+1}}{\Xi_t}\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{P_t}{P_{t+1}}\right]=\mathbb{E}_t\left[Q_{t,t+1}\right]=\frac{1}{R_t^B},\tag{2.3}$$

where $R_t^B=1/\mathbb{E}_t[Q_{t,t+1}]$ is the one-period nominal interest rate faced by the household. Their first-order condition for labor supply satisfies

$$\frac{W_t}{P_tC_t^{\sigma}}=N_t^{\phi}.\tag{2.4}$$

2.1.2. Consumption allocation

Households consume both home (H) and foreign (F) goods:

$$C_t=\Phi C_{Ht}^{\nu/2}C_{Ft}^{1-\nu/2},\tag{2.5}$$

where $\Phi=(\frac{\nu}{2})^{\frac{\nu}{2}}(1-\frac{\nu}{2})^{1-\frac{\nu}{2}}$. Households have a preference bias for domestic goods: they allocate $\nu/2$ share of their expenditure to domestic goods and $1-\nu/2$ to imported goods, where $1<\nu\le 2$. The demand curves are

$$C_{Ht}=\frac{\nu}{2}\frac{P_t}{P_{Ht}}C_t\tag{2.6}$$

$$C_{Ft}=\left(1-\frac{\nu}{2}\right)\frac{P_t}{P_{Ft}}C_t,\tag{2.7}$$

and the CPI aggregates over prices for homes goods and foreign goods:

$$P_t=P_{Ht}^{\nu/2}P_{Ft}^{1-\nu/2}.\tag{2.8}$$

C_{Ht} is a Dixit-Stiglitz aggregator over differentiated home goods:

$$C_{Ht}=\left(\int_0^1 C_{Ht}(i)^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}},\tag{2.9}$$

where the elasticity of substitution $\theta>1$. The demand curve for each differentiated good i is

$$\frac{C_{Ht}(i)}{C_{Ht}}=\left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta},$$

where the producer price index (PPI) is

$$P_{Ht}=\left[\int_0^1 P_{Ht}(i)^{1-\theta}di\right]^{\frac{1}{1-\theta}}.$$

2.2. Inflation, terms of trade, and exchange rate

Inflation: The CPI and PPI inflations are

$$\Pi_t=\frac{P_t}{P_{t-1}}\tag{2.10}$$

$$\Pi_{Ht}=\frac{P_{Ht}}{P_{H,t-1}}.\tag{2.11}$$

Terms of trade: The terms of trade are defined as the price of foreign goods relative to domestic goods:

$$\mathcal{T}_t=\frac{P_{Ft}}{P_{Ht}}.\tag{2.12}$$

In the log-linear form, the terms of trade can be expressed in terms of the real interest rate differential:

$$\tau_t=\mathbb{E}_t\left[\sum_{k=0}^{\infty}(rr_{t+k}^*-rr_{t+k})\right],\tag{2.13}$$

where lowercase letters denote logs: $\tau_t=\log(\mathcal{T}_t)$. $rr_t=r_t^B-\mathbb{E}_t[\pi_{H,t+1}]$ is the home real interest rate, where $r_t^B=\log(R_t^B)$ and $\pi_{Ht}=\log(\Pi_{Ht})$, and rr_t^* is the foreign real interest rate. Derivation details can be found in Appendix A.

Exchange rate: The law of one price holds

$$\mathscr{E}_t=\frac{P_{Ht}}{P_{Ht}^*}=\frac{P_{Ft}}{P_{Ft}^*},\tag{2.14}$$

where $\mathscr{E}_t$ is the nominal exchange rate, which is completely flexible. The exchange rate and the terms of trade are related by

$$\mathcal{T}_t=\mathscr{E}_t\frac{P_{Ft}^*}{P_{Ht}}.\tag{2.15}$$

Complete international financial markets with international risk sharing implies

$$\frac{\Xi_t}{C_t^{\sigma}}=\frac{\Xi_t^*}{(C_t^*)^{\sigma}}\frac{P_t}{\mathscr{E}_tP_t^*}=\frac{\Xi_t^*}{(C_t^*)^{\sigma}}\mathcal{T}_t^{1-\nu}.\tag{2.16}$$

2.3. Firms

There is a continuum of firms, indexed by $i\in[0,1]$. Each firm hires labor and produces differentiated goods with the production function:

$$Y_t(i)=A_tN_t(i),\tag{2.17}$$

where A_t is the exogenous technology process and it follows $\log(A_t)-\log(A)=\rho_A[\log(A_{t-1})-\log(A)]+\varepsilon_{at}$, where $\log(A)$ is the steady-state value and $\varepsilon_{at}\sim N(0,\sigma_a^2)$. Firms' real marginal cost is

$$MC_t=\frac{(1-g)W_t}{A_tP_{Ht}},\tag{2.18}$$

where g is the wage subsidy for firms to ensure the efficient output level at the steady state.

Firms set prices for differentiated goods in the Calvo fashion. A firm can reset its price with probability $1-\zeta$ in each period. When it does, it chooses $\tilde{P}_{Ht}$ to maximize

$$\mathbb{E}_t\sum_{k=0}^{\infty}\zeta^kQ_{t,t+k}Y_{t+k}(i)\left(\tilde{P}_{Ht}-P_{H,t+k}MC_{t+k}\right),$$

subject to the demand curve

$$Y_{t+k}(i)=\left(\frac{\tilde{P}_{Ht}}{P_{H,t+k}}\right)^{-\theta}Y_{t+k},$$

where the stochastic discount factor is $Q_{t,\ t+k}=Q_{t,\ t+1}Q_{t+1,\ t+2}\cdots Q_{t+k-1,\ t+k}$, and the aggregate output is $Y_t=[\int_0^1 Y_t(i)^{1-\frac{1}{\theta}}di]^{\frac{\theta}{\theta-1}}$. The reset price satisfies

$$\tilde{P}_{Ht}=\frac{\theta}{\theta-1}\frac{\mathbb{E}_t\sum_{k=0}^{\infty}Q_{t,t+k}\zeta^k(1-g)W_{t+k}P_{H,t+k}^{\theta}Y_{t+k}/A_{t+k}}{\mathbb{E}_t\sum_{k=0}^{\infty}Q_{t,t+k}\zeta^kP_{H,t+k}^{\theta}Y_{t+k}}.\tag{2.19}$$

Firms keep prices constant when they cannot reoptimize. Finally, the PPI evolves according to

$$P_{Ht} = \left[(1-\zeta)\tilde{P}_{Ht}^{1-\theta} + \zeta P_{H,t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{2.20}$$

2.4. Market clearing and welfare

The goods market-clearing condition is

$$Y_t = C_{Ht} + C^*_{Ht}. \tag{2.21}$$

The labor market clears when

$$N_t = \int_0^1 N_t(i)di. \tag{2.22}$$

Welfare W is defined as the second-order approximation of households' lifetime utility. Adding two countries together, the world welfare is

$$W^W = W + W^*. \tag{2.23}$$

2.5. Monetary policy and the effective lower bound

The monetary policy follows a Taylor interest-rate rule:

$$\hat{s}_t = \rho_s \hat{s}_{t-1} + (1-\rho_s)\left[\phi_\pi \hat{\pi}_{Ht} + \phi_y x_t\right], \tag{2.24}$$

where s_t is the desired interest rate, which is the interest rate implied by the Taylor rule. Hatted variables are log deviations from the steady states $\hat{s}_t = s_t - s$ and $\hat{\pi}_{Ht} = \pi_{Ht} - \pi$, $\pi = \log(\Pi)$, and s and Π are the steady-state nominal interest rate and inflation. $x_t = y_t - y^n_t$ is the output gap, $y_t = \log(Y_t)$, and $y^n_t = \log(Y^n_t)$ is the natural level of output, or the equilibrium output under flexible prices when $\zeta = 0$; see more details in Appendix A.1. ρ_s captures the persistence of the interest-rate rule, and ϕ_π and ϕ_y are the sensitivities of the nominal interest rate to inflation and output, respectively.

2.5.1. Effective lower bound and unconventional monetary policy

During normal times, the policy rate is

$$r_t = s_t.$$

When the ELB binds $s_t < 0$,[3] the policy rate $r_t = 0$. We conveniently summarize all monetary policy actions with the shadow rate $\mathcal{S}_t$:

$$\mathcal{S}_t = \lambda s_t. \tag{2.25}$$

The case of $\lambda = 0$ and $r^B_t = \mathcal{S}_t = r_t = 0$ corresponds to the ELB in the standard New Keynesian model. In the background of the New Keynesian model, money is an alternative to bond. Hence, it is undesirable for an agent to hold a bond that pays a negative interest rate. The standard model is mute about the central bank's unconventional monetary policy.

However, a growing literature argues that unconventional monetary policy has a stimulative effect on the economy that is similar to the effect of conventional policy, which implies $\lambda = 1$; for example, see Wu and Xia (2016), Wu and Zhang (2017), Mouabbi and Sahuc (2017), and Debortoli et al. (2016).

Note that, even when $\lambda = 1$, agents still do not hold a bond that pays a negative interest rate, given money as an alternative. So how does a negative $\mathcal{S}_t$ enter the households' problem? Wu and Zhang (2017) argue that at the ELB, the relevant interest rate for economic agents is not the constant policy rate $r_t = 0$. Rather, it should be some private interest rate, r^B_t, in our households' problem. We can relate the private interest rate to the policy rate by

$$r^B_t = r_t + rp_t,$$

where rp_t is the wedge between the two, which we call the risk premium. The risk premium term could potentially capture the term premium for a long term bond, a corporate spread, or a convenience yield. During normal times, monetary policy actions work through r_t, and $rp_t = rp$ is a constant; hence, $r^B_t = r_t + rp$. When the ELB binds, unconventional monetary policy, such as QE, lowers rp_t to further provide stimulus.

As a convenient shortcut, we write

$$r^B_t = \mathcal{S}_t + rp, \tag{2.26}$$

and use the shadow rate $\mathcal{S}_t$ as a summary statistic for all conventional and unconventional monetary policy.[4] Note, $\mathcal{S}_t = s_t = r_t$ during normal times. Therefore, our model is the same as the standard model, albeit the constant premium rp. However, our model differs from the standard model at the ELB because $\mathcal{S}_t$ summarizes the time-varying part of rp_t and incorporates unconventional monetary policy. Although $\mathcal{S}_t$<0, r^B_t remains positive to be consistent with the households' optimizing behavior.[5]

In this paper, we do not limit to $\lambda = 0$ or $\lambda = 1$. Rather, we explore all possibilities along $\lambda \in [0,1]$. Quantitative analyses of the theoretical model are in Section 3, and empirical results follow in Section 4.

3. Anomalies at the ELB and unconventional monetary policy

This section first discusses analytically and quantitatively the anomalies at the ELB artificially created by the standard New Keynesian model, which does not capture any effort by the central bank's unconventional monetary policy; that is, $\lambda = 0$ in (2.25). By contrast, we demonstrate these anomalies disappear once unconventional monetary policy is introduced in our model. Next, we relax the model assumptions in two steps. First, we allow different degrees of activeness for unconventional monetary policy $\lambda \in [0,1]$. Second, we further relax $\lambda = \lambda^*$ and allow the two countries to implement unconventional monetary policy differently, and study their interactions. Finally, we consider several alternative model and parameter specifications.

3.1. Anomalies at the ELB

This section presents the anomalies at the ELB: when a negative supply shock hits the economy, output and terms of trade increase, which is the opposite direction from normal times. These anomalies disappear when unconventional monetary policy is implemented. We first derive some analytical results in a simplified setting to gain some intuition, and then relax the simplifying assumptions and present quantitative results.

3.1.1. Analytical results

In this section, we derive analytical properties to provide some intuition with the following simplifying restrictions: $\rho_s = 0$ and $\xi_t = 0\ \forall\ t$, so that for any variable z_t, we can write $\mathbb{E}_t[z_{t+1}] = \rho_a z_t$. The analytical analysis also imposes $\phi_y = 0$ for simplicity. We create the ELB

3 For simplicity, we take 0 as the lower bound, and hence the ELB becomes the zero lower bound. In practice, the lower bound does not necessarily have to be zero (see Wu and Xia (2016)) or a constant (see Wu and Xia (2017)).

4 Wu and Zhang (2017) illustrated that the differences between various private interest rates and Wu and Xia's (2016) shadow rate appear to be constant.

5 Wu and Zhang (2017) demonstrated that various private interest rates in the United States are still positive and not constraint during the ZLB era.

environment with an interest-rate peg at the steady state $\hat{\mathcal{S}}_t = 0$. We find the solution that solves for any generic λ first and then impose λ = 0 for the ELB, and ignore other potential equilibria that only arise at the ELB. We will relax all these assumptions in the quantitative Section 3.1.2.

When a supply shock occurs, the inflation differential, output differential, and terms of trade move with technology as follows:

$$\hat{\pi}_{Ht} - \hat{\pi}^*_{Ft} = -2\Theta(1+\phi)(1-\rho_a)\sigma_0\Lambda_a\hat{a}_t \tag{3.1}$$

$$\hat{y}_t - \hat{y}^*_t = \Theta(1+\phi)(\lambda\phi_\pi - \rho_a)(D+1)\Lambda_a\hat{a}_t \tag{3.2}$$

$$\hat{\tau}_t = \Theta(1+\phi)(\lambda\phi_\pi - \rho_a)\frac{\sigma(D+1)}{D}\Lambda_a\hat{a}_t, \tag{3.3}$$

where $\hat{a}_t = \log(A_t) - \log(A)$, $\hat{y}_t = y_t - y$, $\hat{\tau}_t = \tau_t - \tau$, and $y = \log(Y)$ and $\tau = \log(\mathcal{T})$ are the steady-state values. $\Theta = \frac{(1-\beta\kappa)(1-\kappa)}{\kappa} > 0$, $\Lambda_a = \frac{1}{\Theta(\sigma/D+\phi)(\lambda\phi_\pi-\rho_a)(D+1)+2\sigma_0(1-\rho_a)(1-\beta\rho_a)}$, $D = [(\nu-1)^2 + \sigma\nu(2-\nu)] > 0$, $\sigma_0 = \sigma - (1-\nu/2)(\sigma-1)\nu\sigma/D = \sigma(D+1)/(2D) > 0$. These equations lead to the following proposition:

Proposition 1. If $\phi_\pi > 1$ and $\Lambda_a > 0$, $\hat{a}_t < 0$ implies $\hat{\pi}_{Ht} - \hat{\pi}^*_{Ft} > 0$, and

- $\hat{y}_t - \hat{y}^*_t < 0$, $\hat{\tau}_t < 0$ *when* $\lambda = 1$.
- $\hat{y}_t - \hat{y}^*_t > 0$, $\hat{\tau}_t > 0$ *when* $\lambda = 0$.

Proof: See Appendix B.1.

The contrast between the two cases in Proposition 1 illustrates the anomalies. To demonstrate the intuition, we ignore the home supply shock's foreign effect for now, which we will see is small in the quantitative section. During normal times λ = 1, a negative home TPF shock lowers the domestic output and terms of trade when the monetary policy obeys the Taylor principal $\phi_\pi > 1$. By contrast, when the ELB binds and the central bank is completely out of the picture, the same shock stimulates its own economy by raising equilibrium output, and increases the terms of trade. In our setting, unconventional monetary policy in (2.24) and (2.25) works the same as the conventional Taylor rule when it is fully active with λ = 1. Hence, results for unconventional monetary policy are identical to normal times. $\Lambda_a > 0$ is imposed to guarantee inflation moves in the same direction whether λ = 0 or 1. It is always satisfied for λ = 1, and when λ = 0, it is guaranteed by $0 < \rho_a < \overline{\rho}_a$, where the bound is defined in Appendix B.1.

Proposition 1 makes statements about inflation and output differentials, which we then interpret as home quantities by approximating changes to the foreign economy to zero. To see how the home economy moves without approximation, we'll study the special case of $\sigma = 1$ or $\nu = 2$, in which the home shock does not move across the border. The case $\nu = 2$ corresponds to complete home bias and hence no trade, whereas when $\sigma = 1$, income and substitution effects in the foreign economy are completely canceled out.

Corollary 1. If $(\sigma-1)(\nu-2) = 0$, $\phi_\pi > 1$, and $\Lambda_a > 0$, $\hat{a}_t < 0$ implies $\hat{\pi}^*_{Ft} = 0$, $\hat{y}^*_t = 0$, $\hat{\pi}_{Ht} > 0$, and

- $\hat{y}_t < 0$, $\hat{\tau}_t < 0$ *when* $\lambda = 1$.
- $\hat{y}_t > 0$, $\hat{\tau}_t > 0$ *when* $\lambda = 0$.

Proof: See Appendix B.2.

Corollary 1 illustrates similar anomalies to Proposition 1. The difference is when $\sigma = 1$ or $\nu = 2$, the foreign economy does not move in response to the home TFP shock. Hence, we can make precise statements about the home economy.

Next, we study how international trade contributes to the anomalies at the ELB in Proposition 1. We compare the case with international trade $\nu < 2$ with the no-trade case $\nu = 2$.

Proposition 2. If $\lambda = 0$, $1 < \sigma < \phi$ and $\Lambda_a > 0$, $\hat{a}_t < 0$ implies $\hat{y}_t - \hat{y}^*_t \geq (\hat{y}_t - \hat{y}^*_t)|_{\nu=2} > 0$, and $\hat{\tau}_t \geq \hat{\tau}_t|_{\nu=2} > 0$.

Proof: See Appendix B.3.

With some mild condition between σ and ϕ, international trade amplifies the impact of the TFP shock on output and terms of trade, which makes the anomalies even more prominent.

3.1.2. Quantitative illustration

Setup for quantitative analysis: We set up a quantitative environment here that we will use hereafter, where we relax all the assumptions imposed in Section 3.1.1. We study economies' responses to a negative home TFP shock, which serves as a supply shock. We create the ELB environment with a sequence of preference shocks, and use the occasionally binding method of Guerrieri and Iacoviello (2015) to solve the model. Model details are in Appenidx A, and details for calibration and the solution method are in Appendix C.

Results: Fig. 1 plots impulse response functions of economic quantities to a negative TFP shock in the home country. Green dots are normal times. Red dashed lines represent that the ELB prevails in both countries. The blue solid lines plot what happens when both countries implement unconventional monetary policy following the historical Taylor rule, that is, $\lambda = \lambda^* = 1$. Note the blue solid lines and green dots always overlap each other.

Fig. 1 illustrates the same anomalies as we discussed in Section 3.1.1. In response to the negative supply shock, output and terms of trade decrease during normal times or with unconventional monetary policy, whereas they increase when the ELB is binding and central banks are absent. Additionally, we also see a contrast for foreign output, albeit in a smaller scale given the origin of the shock.

Next, we calculate welfare, which, unlike variables in Fig. 1, are nonlinear objects. We compute the total welfare in the presence of all the shocks. We find the case with ELB and no policy intervention suffers from the largest welfare losses for both the home and foreign economies. The losses become much smaller when unconventional monetary policy is fully active. Note that normal times have slightly higher welfare compared to the case with unconventional monetary policy, because the ELB is created by preference shocks, which introduce some inefficiency.

3.1.3. Mechanism

The basic mechanism that leads to these results consists of two channels. First, it transmits through inflation and the real interest rate, which works the same as in a closed-economy macro model; see Wu and Zhang (2017), for example. A negative supply shock leads to a higher inflation for home goods. During normal times or with unconventional monetary policy, the nominal interest rate increases as a response, which leads to a higher real interest rate. However, at the ELB, the nominal rate does not move, which lowers the real rate. The lower real rate stimulates demand and hence equilibrium output in the home country.

The open-economy model introduces an additional international channel. To illustrate this channel, we plot in Fig. 2 the ELB cases with and without international trade. The red dashed lines are identical to the red dashed lines in Fig. 1. The case without trade is in black solid lines, where $\nu = 2$.

Without trade, the foreign economy does not react to the home shock, which is consistent with Corollary 1. International trade brings this shock across the border into the foreign economy, which in turn further amplifies its effect on the home output: the home country is

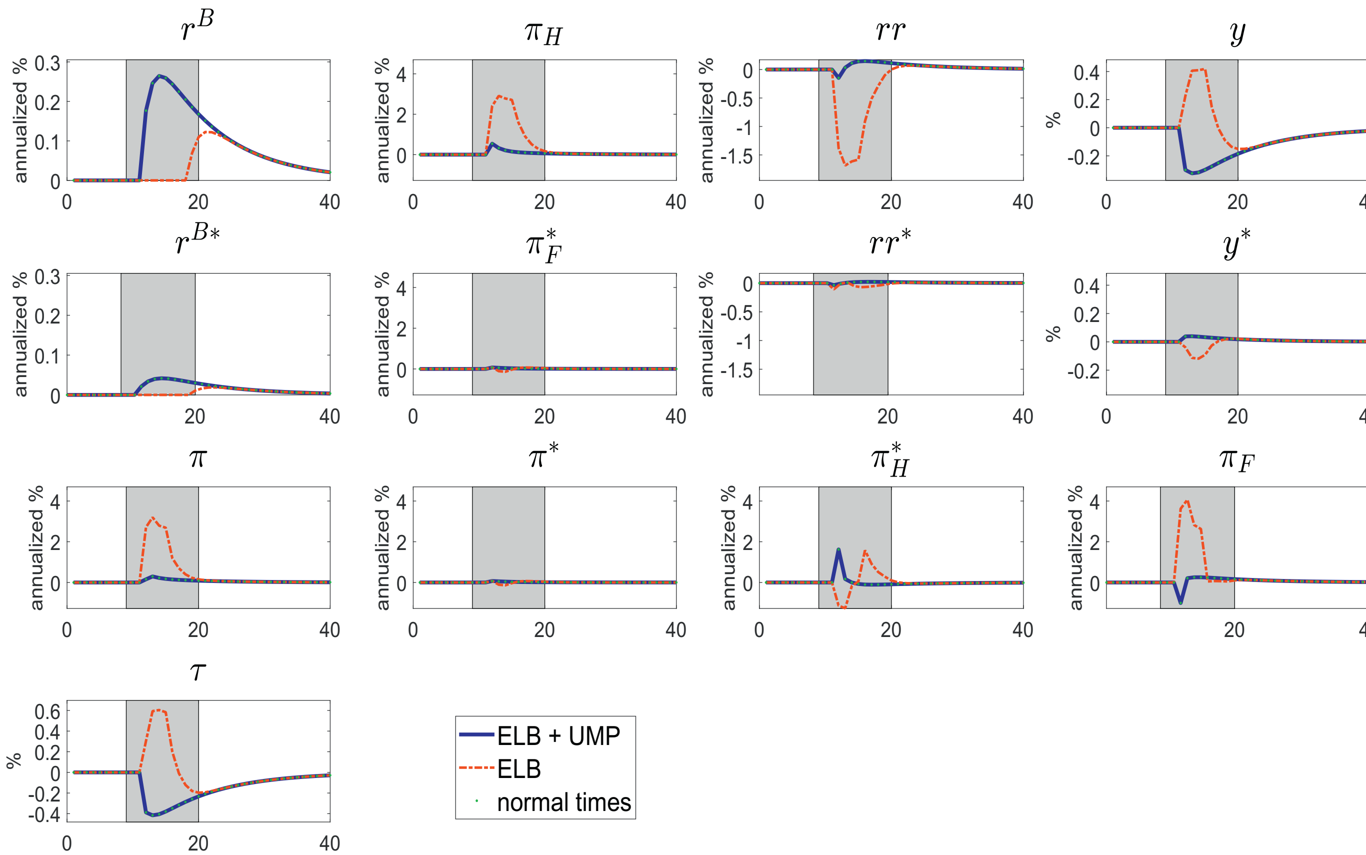

Fig. 1. TFP shock in home country. Notes: A negative technological shock of -0.5% (2 standard deviations) happens in the home country in period 12. To create the ELB environment in the blue solid and red dashed lines, a series of negative preference shocks occurs in both countries in periods 1–15, and the total shock size in each country is 23% (about 2/3 of a standard deviation in each period). We difference out the effect of preference shocks, and only plot the additional effect of the technological shock. The blue solid lines represent the case in which fully active unconventional monetary policy is implemented. The red dashed lines represent the case in which, when the policy rate is bounded by zero, no unconventional monetary policy is implemented. The green dotted lines represent normal times with the standard Taylor rule, and the only shock that hits the economy is the TFP shock. The shaded area marks periods 9–20, for which both countries stay at the ELB with only the preference shocks and without unconventional monetary policy. X-axis: time in quarters; Y-axis: annualized percentage changes for interest rates and inflation, percentage changes relative to the steady states for output and terms of trade.

expected to have a lower real interest rate compared to the foreign country, which increases the terms of trade through (2.13). Higher terms of trade imply a decrease in the relative price of home goods, and therefore households shift their demand toward home goods. Hence, international trade reduces foreign output and further increases home production. This result is consistent with Proposition 2.

Discussion: The anomaly on output is robust across different parameter spaces and alternative model specifications; see Subsection 3.3 for robustness. Garín et al. (forthcoming) and Wieland (forthcoming) discussed a similar anomaly in closed-economy models. The result on terms of trade depends on whether the international financial markets are complete; see further discussion in Section 3.3.4. It is not sensitive to varying parameters or other model specifications though.

3.2. Partially active unconventional monetary policy

We have studied the limiting cases in which unconventional monetary policy is either completely absent, $\lambda = 0$, or fully active, $\lambda = 1$. In this section, we explore all possibilities along $\lambda \in [0,1]$ and allow unconventional monetary policy to be partially active. Section 3.2.1 imposes $\lambda = \lambda^*$ that both central banks' interventions are equally active, and we turn to the asymmetric case in Section 3.2.2.

3.2.1. Symmetric case

Fig. 3 summarizes the response of each economic variable to the TFP shock to a one-dimensional object and plots it as a function of λ, which is the same as λ^*. For the first seven variables, we plot the average impulse response during the ELB. For $\lambda = 0$ ($\lambda = 1$), they are equal to the average values of the red dashed (blue solid) lines from period 12 to 19 (20) in Fig. 1. Both the home output and terms of trade decrease from positive to negative as unconventional monetary policy becomes more active, whereas the foreign output increases from negative to positive. Lifetime welfare, W and W^*, increases when unconventional monetary policy becomes more active, and fully active unconventional monetary policy is the closest to being efficient.

Interestingly, the home country's nominal interest rate does not increase monotonically with λ. Combining (2.24)–(2.26), the nominal rate depends on the products $\lambda\pi_{Ht}$ and λy_t. While λ increases, both inflation and output decrease. For small λ, when λ increases, r_t^B increases. At some point, the rate of decrease in π_{Ht} and y_t overweighs the increase of λ, and hence r_t^B decreases.

3.2.2. Asymmetric case

Next, we further relax the assumption $\lambda = \lambda^*$ and allow two countries to operate their unconventional monetary policy differently, and study their interactions. Fig. 4 plots summary responses to the TFP shock as functions of λ and λ^*. Different colors represent different values for economic quantities, where light green (dark blue) represents higher (lower) values. The 45-degree lines correspond to the symmetric case $\lambda = \lambda^*$ in Fig. 3.

For the home country, mainly its own policy matters: the more active its central bank is in implementing unconventional monetary policy, the lower its output and inflation. It is similar for the terms of trade: a higher λ is associated with smaller terms of trade.

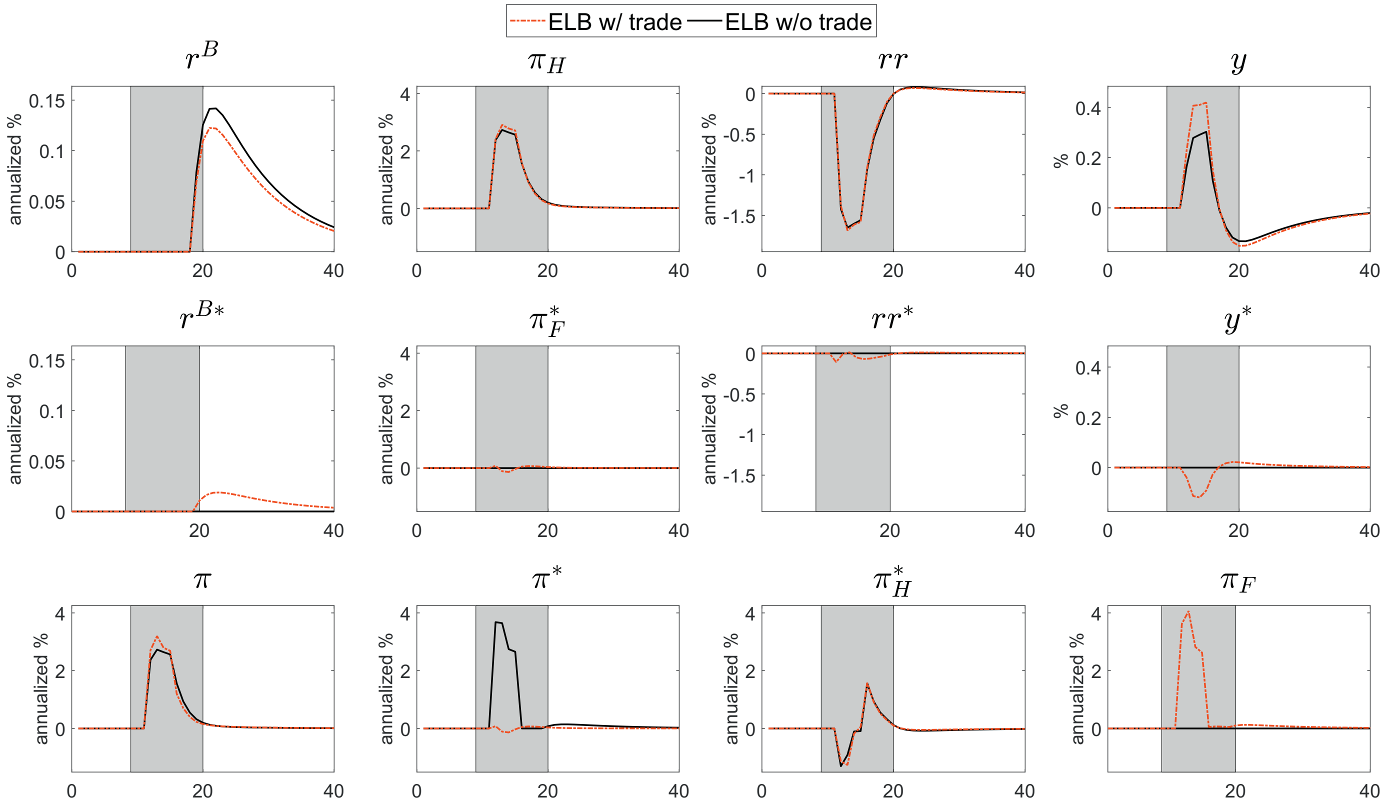

Fig. 2. Trade vs. no trade at the ELB. Notes: A negative technological shock of -0.5% happens in the home country in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1–15, and the total shock size in each country is 23%. We difference out the effect of preference shocks, and only plot the additional effect of the technological shock. The red dashed lines represent the case with trade, and the black solid lines represent the case without trade $\nu = 2$. In both cases, the policy rate is bounded by zero and no unconventional monetary policy is implemented. The shaded area marks periods 9–20, for which both countries stay at the ELB with only the preference shocks and without unconventional monetary policy. X-axis: time in quarters; Y-axis: annualized percentage changes for interest rates and inflations, percentage changes relative to the steady states for output and terms of trade.

The foreign economy, as well as welfare, relies on both central banks. A more active home unconventional policy or a less active foreign policy is associated with higher foreign inflation and output. For welfare, more active home or foreign policy is associated with higher welfare. The most efficient case happens when both countries' policies are fully active.

3.3. Alternative specifications

This section explores alternative specifications and serves as a robustness check. Section 3.3.1 explores alternative parameter spaces. Section 3.3.2 assesses an alternative monetary policy rule, Section 3.3.3 excludes trade, and Section 3.3.4 investigates incomplete financial markets.

3.3.1. Alternative parameter space

This section assesses the robustness of anomalies discussed in Subsection 3.1 across alternative parameter values, where we define anomalies when the maximum response of y and τ are positive at the ELB.[6]

Fig. 5 illustrates the existence of anomalies when we vary the persistence of the TFP dynamics ρ_a, the persistence of the preference shifter ρ_ξ, the inverse of Frisch elasticity of labor supply ϕ, elasticity of intertemporal substitution σ, home bias ν, and the length of preference shocks T_ξ, one at a time, and set other parameters as in the baseline calibration. Gray areas mark that anomalies exist, whereas white areas correspond to the parameter space where anomalies do not appear.

The anomalies are not sensitive to structural parameters ρ_a, ϕ, σ, ν: they exist as long as $\rho_a < 0.98$. This finding is consistent with the condition $0 < \rho_a < \bar{\rho}_a$ in Proposition 1 that guarantees $\Lambda_a > 0$. They always exist for all $\phi \in [0.1,5]$, $\sigma \in [0.1,3]$, and $\nu \in [1,2]$.

Results vary more over parameters related to the preference shock. The gray shades correspond to $0.86 \le \rho_\xi \le 0.9650$ or $T_\xi \ge 12$. Fundamentally, whether anomalies exist depends on how long the ELB lasts,[7] which varies substantially over ρ_ξ and T_ξ. When ρ_ξ is too small or too large or when T_ξ is too small, the number of ELB periods is not large enough to generate anomalies. In the case of ρ_ξ (T_ξ), anomalies are supported if ELB lasts six (three) quarters or longer.

3.3.2. CPI - based Taylor rule

Our baseline specification of the Taylor rule in (2.24) relies on the PPI inflation. A viable alternative is to have the central bank respond to the CPI inflation instead:

$$\hat{s}_t = \rho_s \hat{s}_{t-1} + (1-\rho_s)\left[\phi_\pi \hat{\pi}_t + \phi_y x_t\right]. \tag{3.4}$$

Fig. 6 shows how economic quantities vary with $\lambda = \lambda^*$ when the central bank adopts the alternative Taylor rule. The economies behave similarly to those with the PPI-based rule in Fig. 3. The impulse responses for the domestic economy and the terms of trade are lower if the monetary policy is implemented based on the CPI inflation for most λ, whereas the foreign quantities are higher in this case.

[6] Results for an alternative definition, the average response of y or τ being positive, are very similar.

[7] Garín et al. (forthcoming) draw a similar conclusion.

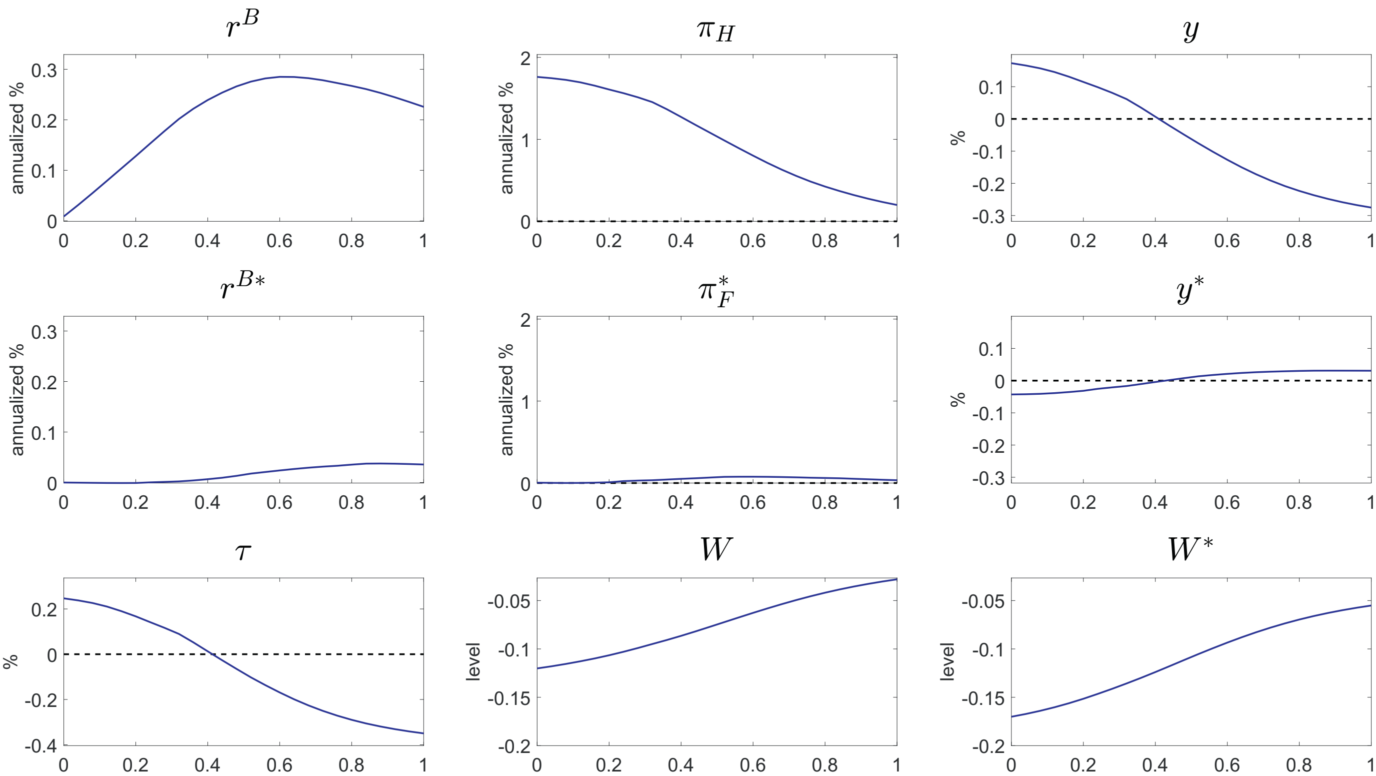

Fig. 3. Varying degrees of activeness of unconventional monetary policy. Notes: For all the variables but W and W^*, we plot the average impulse responses from period 12 to the end of the ELB to the home country's negative TFP shock of -0.5% occurred in period 12 in blue solid lines. Black dashed lines mark zero. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1–15, and the total shock size in each country is 23%. We difference out the effect of preference shocks and only plot the additional effect of the technological shock. W and W^* are the discounted lifetime welfare. X-axis: $\lambda = \lambda^*$. Y-axis: annualized percentage changes for interest rates and inflations, percentage changes relative to the steady states for output and terms of trade, and level for welfare.

3.3.3. No-trade case

Fig. 7 plots the summary responses to the TFP shock as functions of both λ and λ^* for the case with no international trade, which is instrumented by $\nu = 2$. Unlike in Fig. 4, the home economic indicators only move with the home policy indicator λ. The foreign economy in the second row does not move regardless of monetary policy. Welfare, on the other hand, still depends on monetary policies of both countries.

3.3.4. Incomplete financial markets

The benchmark model in Seciton 2 and our analyses thus far assume international financial markets are complete. This section examines incomplete markets. The contrast between the red dashed line and blue solid line in the left panel of Fig. 8 demonstrates the anomaly discussed in Subsection 3.1 still exists for output. See details of the model in Appendix D.

However, whether the financial markets are complete or not does affect trade-related quantities. When the market is complete, terms of trade decrease normally in response to a negative home TFP shock. However, they increase in the setting of incomplete financial markets, which is consistent with what Enders and Müller (2009) find. Moreover, international trade lowers how much output increases at the ELB, mitigating the anomaly.

4. Empirical evidence on unconventional monetary policy

This section empirically investigates unconventional monetary policies at the ELB in the US, Euro area, and UK, and compares them with their corresponding conventional policies. First, we test model implications by comparing impulse responses in a VAR between normal times and the ELB. This exercise allows us to assess whether the anomaly exists in the data. Next, to quantify unconventional monetary policy, we rely on the Taylor rule to compare what has been done with what should have been done.

4.1. Vector autoregression

This section analyzes unconventional monetary policy in a VAR framework. We quantify empirically how output responds to a TFP shock in the US, Euro area, and UK. Then we compare our empirical results with implications from our theoretical model in Section 3 to draw conclusions.

Following Galí and Gambetti (2009), we measure TFP with labor productivity. Our VAR has two variables: the growth rate of labor productivity,[8] $\Delta(y_t - n_t)$, and the log of per-capita hours, n_t. We use a first-order VAR due to the short sample in the quarterly frequency. We identify TFP shocks through the Cholesky decomposition by ordering labor productivity first, which assumes a shock to hours has no contemporaneous impact on labor productivity growth.

We estimate the VAR for the pre-ELB and ELB samples separately. The two samples span from 1985Q2 - 2007Q4[9] and 2009Q1 - 2015Q4 for the US, 1999Q1 - 2011Q3 and 20011Q4 - 2017Q4 for the Euro area, 1993Q1 - 2009Q1 and 2009Q2 - 2017Q4 for the UK. The detailed data sources for the three countries and regions are in Appendix E.

[8] We use the growth rate for stationarity.
[9] We end the pre-ELB sample prior to the Great Recession.

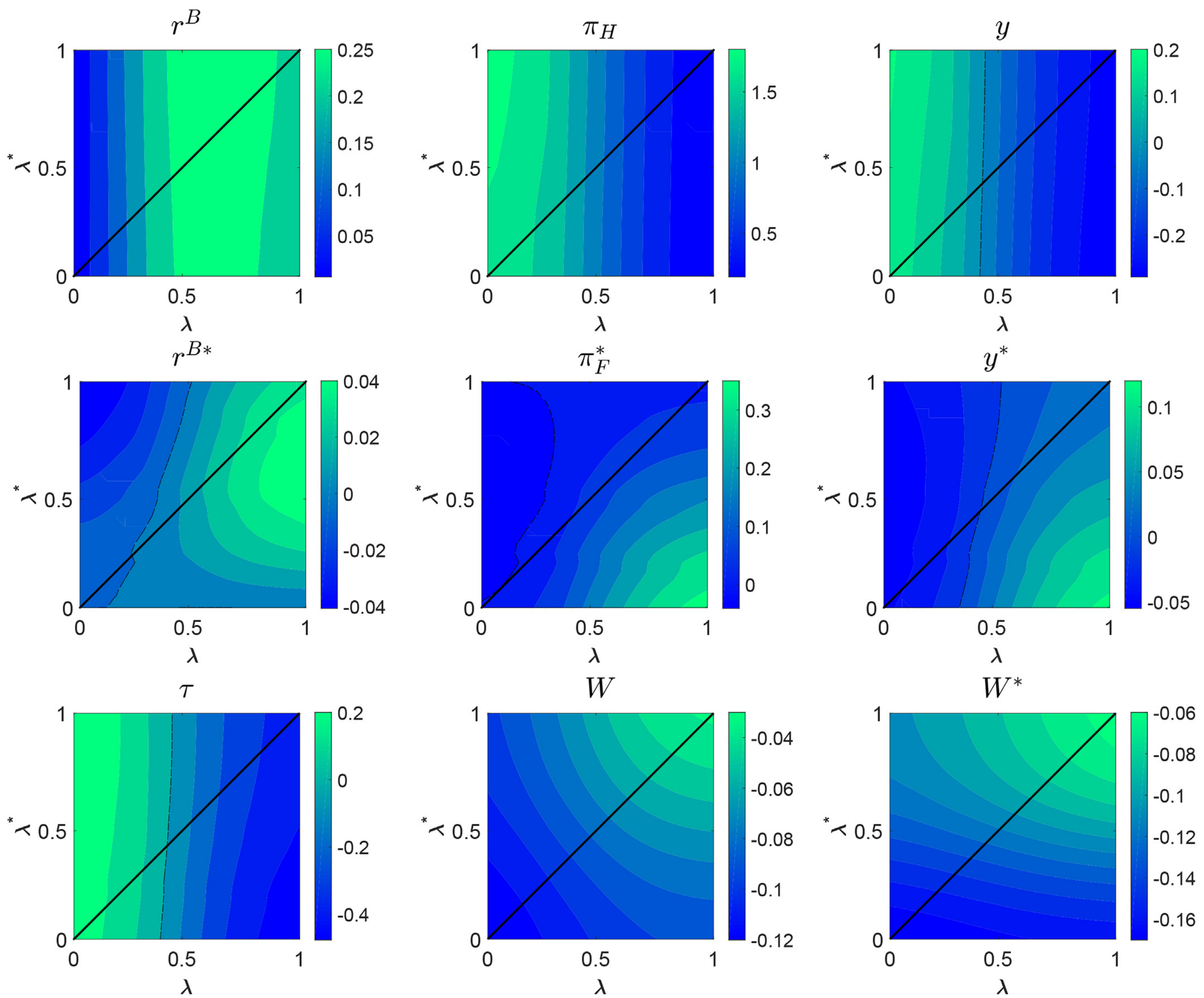

Fig. 4. Asymmetric unconventional monetary policy. Notes: For all the variables but W and W^*, we plot the average impulse responses from period 12 to the end of the ELB to the home country's negative TFP shock of -0.5% in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1–15, and the total shock size in each country is 23%. We difference out the effect of preference shocks and only plot the additional effect of the technological shock. W and W^* are the discounted lifetime welfare. X-axis: λ; Y-axis: λ^*. The color from light green to dark blue corresponds to high to low values. The units are annualized percentage for interest rates and inflation, percentage for output and terms of trade, and level for welfare. The 45-degree lines represent the symmetric case $\lambda = \lambda^*$. The dashed lines are the 0 contours.

Fig. 9 plots impulse responses of output to a -1% shock to labor productivity growth for the three countries and regions.[10] Blue solid lines represent normal times with medians in the thick lines, and 90% confidence intervals in the thin lines. Red dashed lines represent the central tendencies at the ELB. We find that for all three countries and regions, output decreases with a negative TFP shock regardless of normal times or the ELB. This similarity result is in contrast to the anomaly presented by the standard New Keynesian model in Subsection 3.1, and is potential evidence for unconventional monetary policy.

The left panel is for the US. We find the impulse response at the ELB is initially slightly lower than normal times, and then the red dashed and blue thick solid lines track each other closely after five quarters. Moreover, the red dashed line is within the confidence interval in blue. In the case of the US, we do not find anomaly, and our result is consistent with Garín et al. (forthcoming) and Debortoli et al. (2016).[11]

The middle panel is for the Euro area, and the right panel is for the UK. Both of them show that output decreases less at the ELB than in normal times. The differences between normal times and the ELB are statistically significant in both cases, with the UK being more pronounced.

These findings suggest the anomaly does not appear in the data for the three countries and regions we examined. If unconventional monetary policy were the sole source that drives the difference between the standard New Keynesian model and what we see in the data, we would conclude that unconventional monetary policy is as active as usual in the US and is less active for the Euro area and UK, or $\lambda_{US} \approx 1 > \lambda_{Euro} > \lambda_{UK} > 0$.

4.2. Taylor rule

In Subsection 4.1, the VAR qualitatively sorts the effectiveness of unconventional monetary policy among the three regions and countries based on our theoretical model. In this section, we quantify the amount of unconventional monetary policy implemented in each country or

[10] Output is calculated as $y_{t+j} = \sum_{\tau=0}^{j} \Delta(y_{t+\tau} - n_{t+\tau}) + n_{t+j}$, where $y_{t-1} - n_{t-1} = 0$.

[11] We find a similar comparison for Japan. Therefore, the anomaly does not exist in Japan either, which is consistent with Wieland's (forthcoming) findings. The details of the VAR analysis for Japan are in Appendix F.

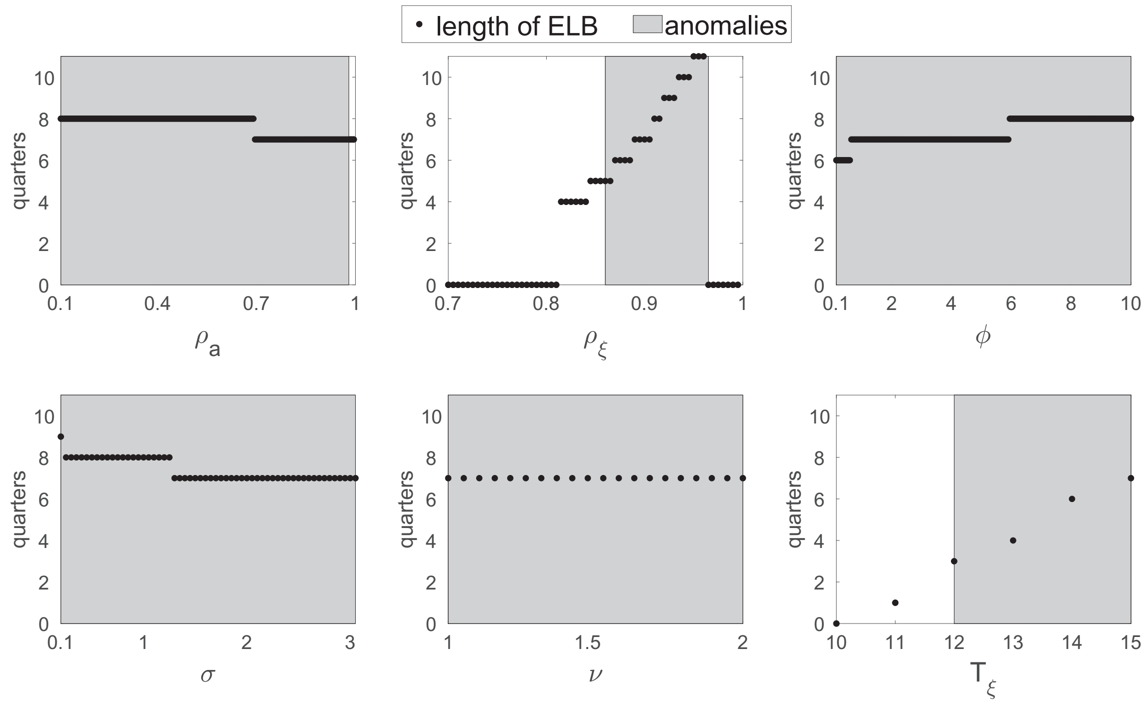

Fig. 5. Anomalies with alternative parameter values. Notes: X-axis: ρ_a in the top left panel, ρ_ξ in the top middle panel, ϕ in the top right panel, σ in the bottom left panel, ν in the bottom middle panel, and T_ξ in the bottom right panel. Y-axis: time in quarters. Black dots: the number of ELB periods after the TFP shock. Gray shades: anomalies exist; white areas: anomalies do not exist.

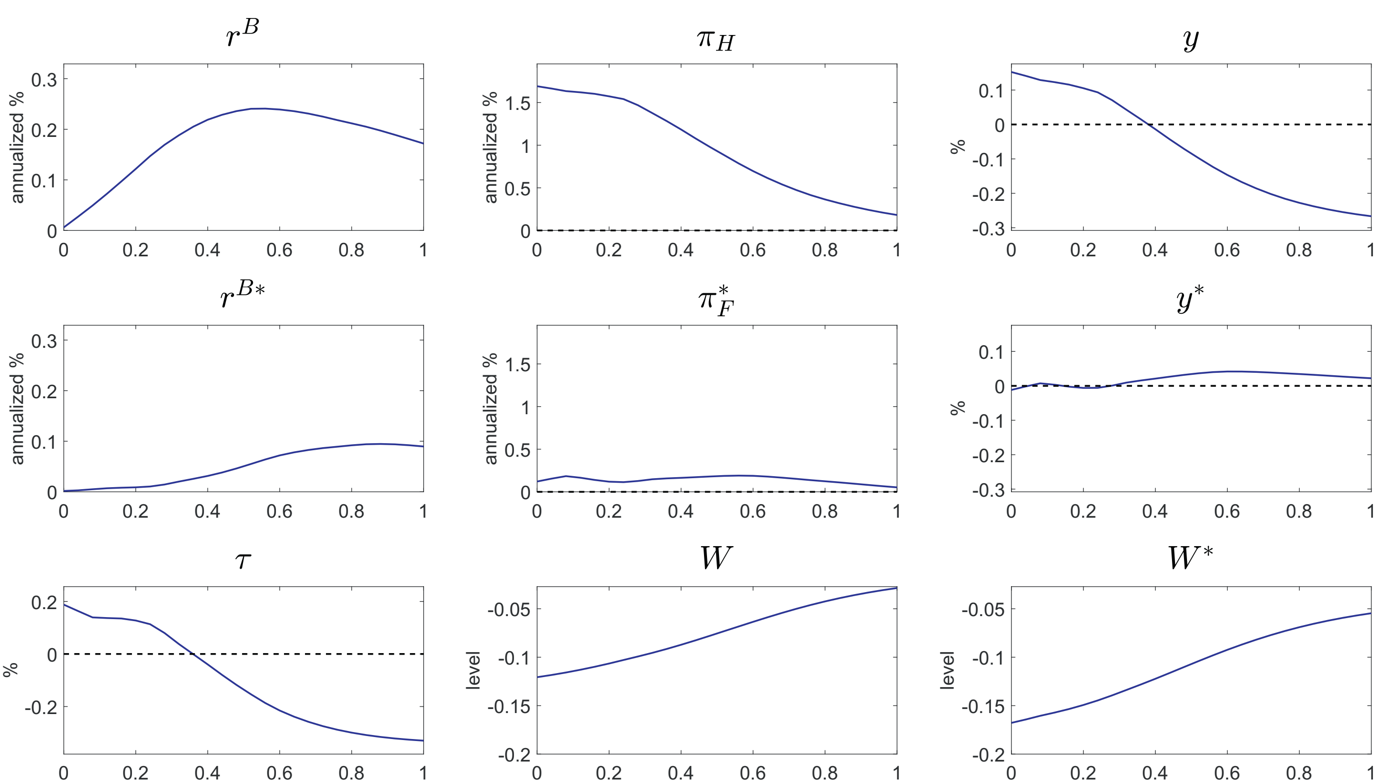

Fig. 6. CPI vs. PPI - based Taylor rule. Notes: For all the variables but W and W^*, we plot the average impulse responses from period 12 to the end of the ELB to the home country's negative TFP shock of -0.5% in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1–15, and the total shock size in each country is 23%. We difference out the effect of preference shocks and only plot the additional effect of the technological shock. W and W^* are the discounted lifetime welfare. X-axis: $\lambda = \lambda^*$.Y-axis: annualized percentage changes for interest rates and inflations, percentage changes relative to the steady states for output and terms of trade, and level for welfare.

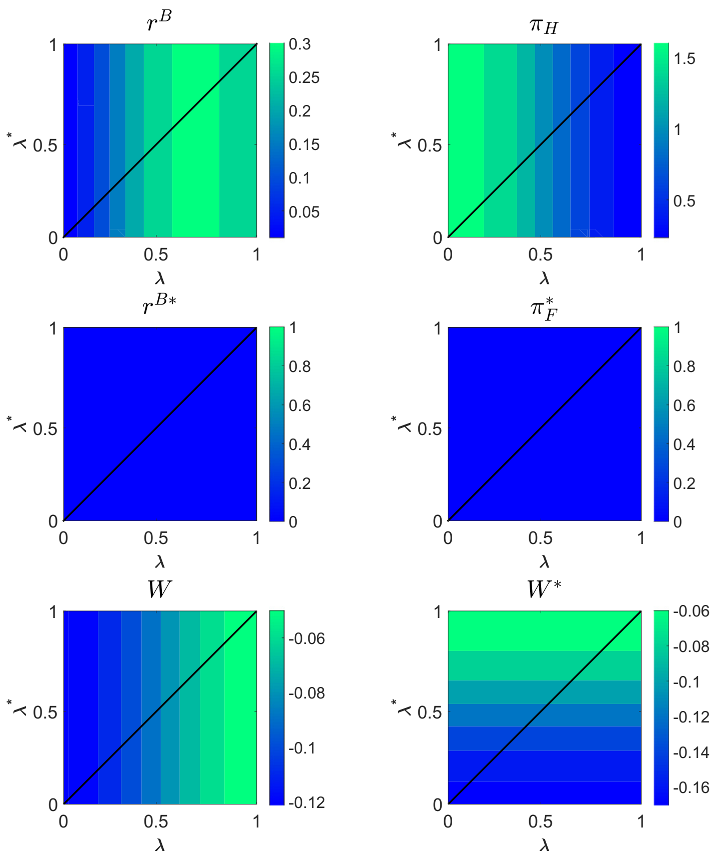

Fig. 7. No-trade case. Notes: For all the variables but W and W^*, we plot the average impulse responses from period 12 to the end of the ELB to the home country's negative TFP shock of -0.5% in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1–15, and the total shock size in each country is 23%. We difference out the effect of preference shocks and only plot the additional effect of the technological shock. W and W^* are the discounted lifetime welfare. X-axis: λ; Y-axis: λ*. The color from light green to dark blue corresponds to high to low values. The units are annualized percentage for interest rates and inflation, percentage for output, and level for welfare. The 45-degree lines represent the symmetric case $\lambda=\lambda^*$. The dashed lines are the 0 contours. The no-trade case is created by $\nu = 2$.

region, and assess whether this amount we observe can explain the difference between the standard New Keynesian model and what we find in our VAR.

We quantify unconventional monetary policy by comparing what has been done at the ELB, measured by the shadow rates of Wu and Xia (2016) and Wu and Xia (2017),[12] with the desired interest rates implied by the historical Taylor rule.

We estimate the historical Taylor rule,

$$r_t = \beta_0 + \beta_1 r_{t-1} + \beta_2 \pi_{Ht} + \beta_3 x_t + \varepsilon_t, \tag{4.1}$$

which is the empirical version of (2.24), via ordinary least squares, using the pre-ELB sample, and label the estimates $\tilde{\beta}_0, \tilde{\beta}_1, \tilde{\beta}_2, \tilde{\beta}_3$. When the ELB is binding, the desired interest rate implied by the historical Taylor rule can be calculated as follows:

$$\tilde{s}_t = \tilde{\beta}_0 + \tilde{\beta}_1 s_{t-1} + \tilde{\beta}_2 \pi_{Ht} + \tilde{\beta}_3 x_t. \tag{4.2}$$

Next, we calculate the activeness of unconventional monetary policy by comparing the implemented monetary policy at the ELB and the desired interest rate. Specifically, we regress the observed shadow rate $\mathcal{S}_t$ on the imputed $\tilde{s}_t$ to get $\tilde{\lambda}$ per (2.25).

Now we turn our attention to s_{t-1} in (4.2). We propose two methods below to proxy it.

[12] Shadow rates are available for the US, the Euro area, and UK, and they can be downloaded from Cynthia Wu's website: https://sites.google.com/view/jingcynthiawu/shadow-rates.

4.2.1. Simple method

The simple method uses the observed shadow rate $\mathcal{S}_{t-1}$ as a proxy for s_{t-1}. Hence, (4.2) becomes

$$\tilde{s}_t = \tilde{\beta}_0 + \tilde{\beta}_1 \mathcal{S}_{t-1} + \tilde{\beta}_2 \pi_{Ht} + \tilde{\beta}_3 x_t. \tag{4.3}$$

The benefit of the simple method is that the shadow rate is observable to us. Hence, the calculation is simple and robust.

4.2.2. Iterative method

To measure s_{t-1} more accurately in the case of small λ, we leverage the relationship in (2.25), and replace (4.2) with

$$\tilde{s}_t = \tilde{\beta}_0 + \tilde{\beta}_1 \mathcal{S}_{t-1}/\tilde{\lambda} + \tilde{\beta}_2 \pi_{Ht} + \tilde{\beta}_3 x_t. \tag{4.4}$$

Now we face a fixed-point problem: (4.4) relies on $\tilde{\lambda}$ to compute $\tilde{s}_t$, whereas to obtain $\tilde{\lambda}$, we regress $\mathcal{S}_t$ on $\tilde{s}_t$. We propose an iterative procedure to solve this fixed-point problem. First, we give an initial guess for λ: $\tilde{\lambda}^{(0)}$. Then we iterate over the following two steps until convergence:

1. Based on $\tilde{\lambda}^{(i)}$, compute $\{\tilde{s}_t^{(i)}\}_{t=1}^T$ using (4.4).
2. Regress $\mathcal{S}_t$ on $\tilde{s}_t^{(i)}$ and compute $\tilde{\lambda}^{(i+1)}$.

4.2.3. Empirical results

We begin with the US. We measure π_{Ht} with the inflation based on the GDP price deflator, x_t is the real GDP minus potential GDP, and r_t corresponds to the effective fed funds rate. The pre-ELB and ELB samples

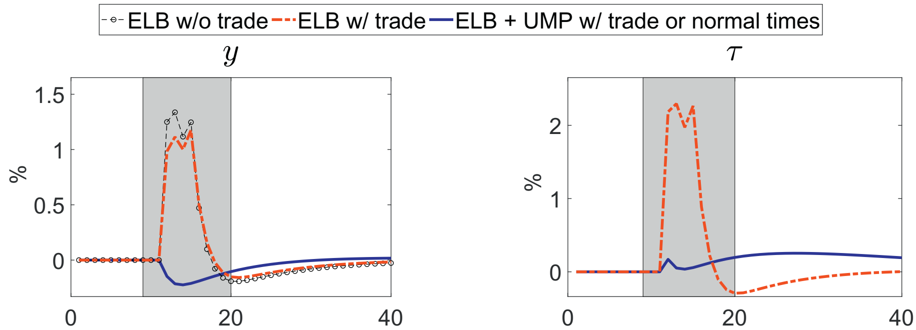

Fig. 8. Incomplete markets. Notes: The international financial markets are incomplete. A negative technological shock of -0.5% happens in the home country in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1 − 15, and the total shock size in each country is 23%. We difference out the effect of preference shocks, and only plot the additional effect of the technological shock. The black dashed line with circles represents the ELB case without trade, the red dashed lines represent the case with trade, and the blue solid lines represent the case with trade and UMP, which is the same as in normal times. The shaded area marks periods 9 - 20, for which both countries stay at the ELB with only the preference shocks and without unconventional monetary policy. X-axis: time in quarters; Y-axis: percentage changes relative to the steady states.

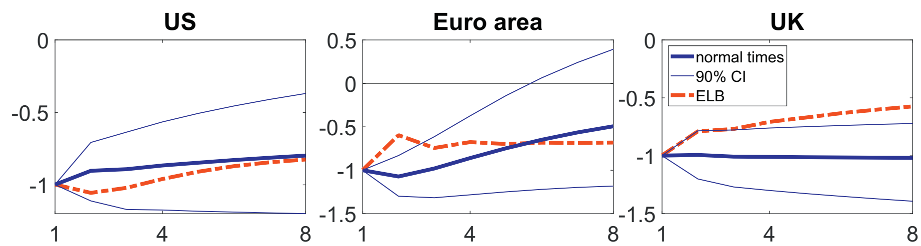

Fig. 9. Impulse response of output to a productivity shock. Notes: Impulse responses of output to a -1% shock to labor productivity growth. The blue solid lines are normal times, with thick lines being medians and thin lines representing 90% confidence intervals. The red dashed lines are the median impulse responses at the ELB. X-axis: time in quarters; Y-axis: percentage changes in output.

are the same as in Subsection 4.1. The details of the data are in Appendix E. The estimate of the simple method is 1.02, and is 1.12 from the iterative method. We conclude that the US unconventional monetary policy is as active as, if not more active than, the historical Taylor rule.

The Taylor rule is known to vary over different sample periods, and researchers' choices of sample periods in the literature are far from unanimous. We quantify the variation of our estimates by varying the pre-ELB estimation sample: the beginning of the sample ranges from $t_0 \in \{1982Q1 : 1990Q1\}$ and the end of the sample varies from $t_1 \in \{2003Q1 : 2008Q4\}$, which covers the majority of popular choices. We compute a λ for each combination of t_0 and t_1 and plot its distribution across all possible combinations in Fig. 10. The left panel plots the distribution for the simple method, and the right panel uses the iterative method. They both center around 1: the median for the simple method is 1.03, and is 1.19 for the iterative method. The standard error for the simple method is 0.065, and is 0.45 for the iterative method. The iterative method displays a larger variation across different sample periods than the simple method. On the other hand, the results from the simple method might be biased if λ is far from 1. This is the classic bias-variance tradeoff.

For the Euro area and UK, quarterly real potential GDP is not available. Hence, we replace x_t in (4.1) with output growth Δy_t, measured by the growth rate of real GDP. The r_t for the Euro area is the 3-month Euro Interbank Offered Rate (Euribor), and it is the Bank of England

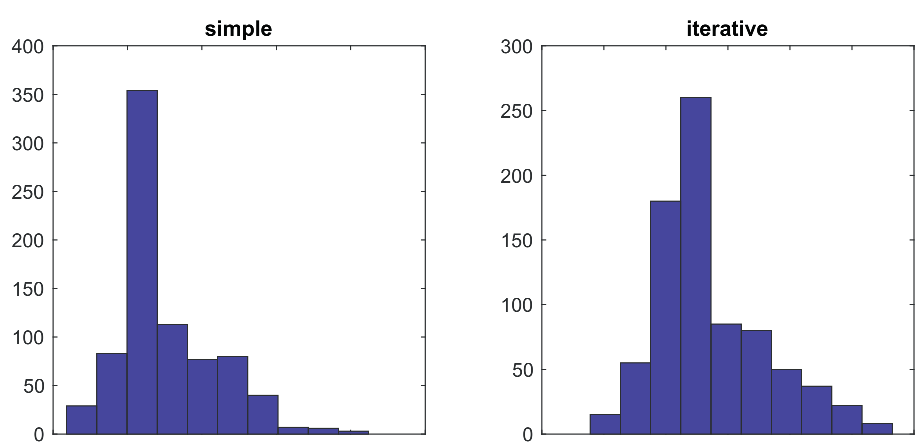

Fig. 10. Distribution of λ for the US. Notes: $t_0 \in \{1982Q1 : 1990Q1\}$, and $t_1 \in \{2003Q4 : 2008Q4\}$. For each combination of t_0 and t_1, estimate a λ from t_0 to t_1. Then plot the distribution across all possible combinations of t_0 and t_1. Left panel: simple method; right panel: iterative method.

policy rate for the UK. The details of the data are in Appendix E. For the Euro area, $t_0 \in \{1998Q2 : 1999Q1\}$ and $t_1 \in \{2009Q1 : 2011Q3\}$. The ELB period is from $t_1 + 1$ to 2017Q4. The median estimate for the iterative (simple) method is 0.63 (0.985) with a standard error of 1.07 (0.031). For the UK, $t_0 \in \{1993Q1 : 2003Q1\}$, $t_1 = 2009Q1$, and the ELB period is from 2009Q2 to 2017Q4. The median from the iterative (simple) method is 0.39 (0.98), with a standard error of 4.10 (0.10). Note the variations across samples are larger for the Euro area - especially for the UK - than for the US, partly due to a shorter sample.

In summary, all three central banks have implemented a considerable amount of unconventional monetary policy: the US operates following the historical Taylor rule, the Euro area and UK's unconventional monetary policies are less active, or $\lambda_{US} \approx 1 > \lambda_{Euro} > \lambda_{UK} > 0$, which is consistent with what we find in Subsection 4.1.

5. Conclusion

We have introduced a new open-economy New Keynesian model. Our model provides a tractable framework that allows for unconventional monetary policy when the ELB is binding. We find when unconventional monetary policy operates following the historical Taylor rule, the anomalies in a standard model, namely, that output and terms of trade increase in response to a negative supply shock, disappear. Our model allows unconventional policy to be partially active and potentially asymmetric between the two countries. Empirically, we assess unconventional monetary policy across the US, Euro area, and UK. The VAR analysis and the Taylor rule together point to the conclusion: The US has operated its unconventional monetary policy following the historical Taylor rule. Although both the Euro area and UK have also implemented a considerable amount of unconventional policies, they have done less than what they normally would have.

Appendix A. Model appendix

A.1. Flexible-price equilibrium

For any variable Z_t, Z_t^n represents its flexible-price counterpart. In the flexible-price economy, real marginal cost is a constant

$$\frac{\theta-1}{\theta} \equiv MC_t^n = \frac{(1-g)\left(W_t^n/P_{Ht}^n\right)}{A_t} = \frac{(1-g)\left(W_t^n/P_t^n\right)\left(\mathcal{T}_t^n\right)^{1-\nu/2}}{A_t}, \tag{A.1}$$

and the optimal wage subsidy satisfies $\frac{\theta-1}{\theta} = 1-g$. Combining the labor-supply condition, $(C_t^n)^\sigma (N_t^n)^\phi = W_t^n/P_t^n$, the production function, $Y_t^n = A_t N_t^n$, and the real marginal cost (A.1), we have

$$\left(C_t^n\right)^\sigma \left(N_t^n\right)^\phi = A_t \left(\mathcal{T}_t^n\right)^{\nu/2-1}. \tag{A.2}$$

The risk-sharing condition holds as follows:

$$\frac{\Xi_t}{\left(C_t^n\right)^\sigma} = \frac{\Xi_t^*}{\left(C_t^{n*}\right)^\sigma} \frac{P_t^n}{\mathcal{E}_t^n P_t^{n*}} = \frac{\Xi_t^*}{\left(C_t^{n*}\right)^\sigma} \left(\mathcal{T}_t^n\right)^{1-\nu}. \tag{A.3}$$

The market-clearing conditions are

$$Y_t^n = \frac{\nu}{2} \left(\mathcal{T}_t^n\right)^{1-\nu/2} C_t^n + \left(1-\frac{\nu}{2}\right) \left(\mathcal{T}_t^n\right)^{\nu/2} C_t^{n*} \tag{A.4}$$

$$Y_t^{n*} = \frac{\nu}{2} \left(\mathcal{T}_t^n\right)^{-1+\nu/2} C_t^{n*} + \left(1-\frac{\nu}{2}\right) \left(\mathcal{T}_t^n\right)^{-\nu/2} C_t^n. \tag{A.5}$$

A.2. Log-linearized equations

Log-linearizing the consumption-savings decision in (2.3) and its foreign counterpart yields[13]

$$\hat{r}_t^B - \mathbb{E}_t\left[-\left(\hat{\xi}_{t+1} - \hat{\xi}_t\right) + \sigma\left(\hat{c}_{t+1} - \hat{c}_t\right) + \hat{\pi}_{t+1}\right] = 0 \tag{A.6}$$

$$\hat{r}_t^{B*} - \mathbb{E}_t\left[-\left(\hat{\xi}_{t+1}^* - \hat{\xi}_t^*\right) + \sigma\left(\hat{c}_{t+1}^* - \hat{c}_t^*\right) + \hat{\pi}_{t+1}^*\right] = 0. \tag{A.7}$$

The labor-supply decision in (2.4) becomes

$$\hat{w}_t = \sigma \hat{c}_t + \phi \hat{n}_t \tag{A.8}$$

$$\hat{w}_t^* = \sigma \hat{c}_t^* + \phi \hat{n}_t^*. \tag{A.9}$$

The market-clearing condition in (2.21) becomes

$$\hat{y}_t = \left[\frac{\nu}{2}\hat{c}_t + \left(1-\frac{\nu}{2}\right)\hat{c}_t^*\right] + \nu\left(1-\frac{\nu}{2}\right)\hat{\tau}_t \tag{A.10}$$

$$\hat{y}_t^* = \left[\frac{\nu}{2}\hat{c}_t^* + \left(1-\frac{\nu}{2}\right)\hat{c}_t\right] - \nu\left(1-\frac{\nu}{2}\right)\hat{\tau}_t. \tag{A.11}$$

The international risk-sharing condition (2.16) is

$$\hat{\xi}_t - \sigma\hat{c}_t = \hat{\xi}_t^* - \sigma\hat{c}_t^* + p_t - e_t - p_t^* = \hat{\xi}_t^* - \sigma\hat{c}_t^* + (1-\nu)\hat{\tau}_t. \tag{A.12}$$

The production function in (2.17) becomes

$$\hat{y}_t = \hat{a}_t + \hat{n}_t \tag{A.13}$$

$$\hat{y}_t = \hat{a}_t^* + \hat{n}_t^*. \tag{A.14}$$

Combining (2.18) and the labor-supply decision (2.4) results in the real marginal costs:

$$\widehat{mc}_t = \phi\hat{n}_t + \sigma\hat{c}_t - \hat{a}_t + (1-\nu/2)\hat{\tau}_t \tag{A.15}$$

$$\widehat{mc}_t^* = \phi\hat{n}_t^* + \sigma\hat{c}_t^* - \hat{a}_t^* - (1-\nu/2)\hat{\tau}_t. \tag{A.16}$$

The CPI price in (2.8) yields to

$$p_t = \frac{\nu}{2} p_{Ht} + \left(1-\frac{\nu}{2}\right) p_{Ft} \tag{A.17}$$

$$p_t^* = \frac{\nu}{2} p_{Ft}^* + \left(1-\frac{\nu}{2}\right) p_{Ht}^*. \tag{A.18}$$

The definitions of CPI (2.10) and PPI inflation (2.11) are

$$\hat{\pi}_t = p_t - p_{t-1} \tag{A.19}$$

$$\hat{\pi}_t^* = p_t^* - p_{t-1}^* \tag{A.20}$$

$$\hat{\pi}_{Ht} = p_{Ht} - p_{H,t-1} \tag{A.21}$$

$$\hat{\pi}_{Ft} = p_{Ft} - p_{F,t-1}. \tag{A.22}$$

Combining (2.18), (2.19), and (2.20), the dynamics for the PPI inflation can be written as

$$\hat{\pi}_{Ht} = \beta\mathbb{E}_t\hat{\pi}_{H,t+1} + \Theta\widehat{mc}_t \tag{A.23}$$

$$\hat{\pi}_{Ft}^* = \beta\mathbb{E}_t\hat{\pi}_{F,t+1}^* + \Theta\widehat{mc}_t^*, \tag{A.24}$$

[13] We will omit “log-linearize” and “foreign counterpart” hereafter for brevity.

where $\Theta = \frac{(1-\beta\zeta)(1-\zeta)}{\zeta} > 0$. The definitions for terms of trade (2.12) and nominal exchange rate (2.14) are

$$\hat{\tau}_t = p_{Ft} - p_{Ht} \tag{A.25}$$

$$p_{Ht} = e_t + p^*_{Ht} \tag{A.26}$$

$$p_{Ft} = e_t + p^*_{Ft}. \tag{A.27}$$

Combining (A.19) – (A.22) and (A.25) – (A.27), the CPI inflation can be expressed as a function of PPI inflation and terms of trade:

$$\hat{\pi}_t = \hat{\pi}_{Ht} + \left(1 - \frac{\nu}{2}\right)\Delta\hat{\tau}_t \tag{A.28}$$

$$\hat{\pi}^*_t = \hat{\pi}^*_{Ft} - \left(1 - \frac{\nu}{2}\right)\Delta\hat{\tau}_t. \tag{A.29}$$

The labor-supply decision (A.2) in the flexible-price economy becomes

$$\sigma\hat{c}^n_t + \phi\hat{n}^n_t = \hat{a}_t + (\nu/2 - 1)\hat{\tau}^n_t \tag{A.30}$$

$$\sigma\hat{c}^{n*}_t + \phi\hat{n}^{n*}_t = \hat{a}^*_t - (\nu/2 - 1)\hat{\tau}^n_t. \tag{A.31}$$

The international risk-sharing condition (A.3) in the flexible-price economy is

$$\sigma\left(\hat{c}^n_t - \hat{c}^{n*}_t\right) - \left(\hat{\xi}_t - \hat{\xi}^*_t\right) - (\nu - 1)\hat{\tau}^n_t = 0. \tag{A.32}$$

The market-clearing conditions (A.4) and (A.5) in the flexible-price economy are

$$\hat{y}^n_t = \left[\frac{\nu}{2}\hat{c}^n_t + \left(1 - \frac{\nu}{2}\right)\hat{c}^{n*}_t\right] + \nu\left(1 - \frac{\nu}{2}\right)\hat{\tau}^n_t \tag{A.33}$$

$$\hat{y}^{n*}_t = \left[\frac{\nu}{2}\hat{c}^{n*}_t + \left(1 - \frac{\nu}{2}\right)\hat{c}^n_t\right] - \nu\left(1 - \frac{\nu}{2}\right)\hat{\tau}^n_t. \tag{A.34}$$

The output gaps are defined as

$$x_t = y_t - y^n_t \tag{A.35}$$

$$x^*_t = y^*_t - y^{n*}_t. \tag{A.36}$$

Eqs. (A.6) to (A.36) and the monetary policy rules (2.24) and (2.25) and their foreign counterparts summarize all equilibrium conditions.

A.3. Exchange rate, terms of trade, and interest rates

Combining the Euler (A.6) and (A.7) with the international risk-sharing condition (A.12), we obtain

$$\mathbb{E}_t[\Delta e_{t+1}] = r^B_t - r^{B*}_t. \tag{A.37}$$

Combining (A.37), (A.25), and (A.27), we get

$$\begin{aligned}\hat{\tau}_t &= \left(\hat{r}^{B*}_t - \mathbb{E}_t\left[\hat{\pi}^*_{F,t+1}\right]\right) - \left(\hat{r}^B_t - \mathbb{E}_t\left[\hat{\pi}_{H,t+1}\right]\right) + \mathbb{E}_t[\hat{\tau}_{t+1}] \\ &= \widehat{rr}^*_t - \widehat{rr}_t + \mathbb{E}_t[\hat{\tau}_{t+1}].\end{aligned} \tag{A.38}$$

Solving (A.38) forward under the stationarity condition $\lim_{k\to\infty}\mathbb{E}_t \hat{\tau}_{t+k} = 0$, we obtain (2.13).

Appendix B. Proofs

B.1. Proof of Proposition 1

In Appendix B.1 to Appendix B.3, there is only a home country's TFP shock, that is, $\hat{a}^*_t = \hat{\xi}_t = \hat{\xi}^*_t = 0$. Combining the market-clearing conditions (A.10) and (A.11) and the international risk-sharing condition (A.12), terms of trade can be expressed as a function of relative output:

$$\hat{\tau}_t = \frac{\sigma}{D}\left(\hat{y}_t - \hat{y}^*_t\right). \tag{B.1}$$

Combining the Euler eqs. (A.6) and (A.7) with the market-clearing conditions (A.10) and (A.11), international risk sharing (A.12), and the definition of terms of trade (A.25), we get the IS curves for the home and foreign countries:

$$\hat{y}_t = \mathbb{E}_t\hat{y}_{t+1} - \frac{1}{\sigma_0}\left(\hat{r}^B_t - \mathbb{E}_t\hat{\pi}_{H,t+1}\right) + K_2\left(\mathbb{E}_t\hat{y}^*_{t+1} - \hat{y}^*_t\right) \tag{B.2}$$

$$\hat{y}^*_t = \mathbb{E}_t\hat{y}^*_{t+1} - \frac{1}{\sigma_0}\left(\hat{r}^{B*}_t - \mathbb{E}_t\hat{\pi}^*_{F,t+1}\right) + K_2\left(\mathbb{E}_t\hat{y}_{t+1} - \hat{y}_t\right), \tag{B.3}$$

where $\sigma_0 = \sigma - K_1$, $K_1 = (1 - \nu/2)(\sigma - 1)\nu\sigma/D = \frac{\sigma}{2}\frac{D-1}{D}$, $D = [(\nu - 1)^2 + \sigma\nu(2 - \nu)]$, and $K_2 = K_1/\sigma_0$. Take the difference between the home and foreign IS curves,

$$\begin{aligned}&\left(\hat{r}^B_t - \hat{r}^{B*}_t\right) - \mathbb{E}_t\left(\hat{\pi}_{H,t+1} - \hat{\pi}^*_{F,t+1}\right) \\ &= \sigma_0(1 - K_2)\mathbb{E}_t\left[\left(\hat{y}_{t+1} - \hat{y}^*_{t+1}\right) - \left(\hat{y}_t - \hat{y}^*_t\right)\right].\end{aligned} \tag{B.4}$$

The monetary policy rules are

$$\hat{\mathcal{S}}_t = \lambda\phi_\pi\hat{\pi}_{Ht} \tag{B.5}$$

$$\hat{\mathcal{S}}^*_t = \lambda\phi_\pi\hat{\pi}^*_{Ft}, \tag{B.6}$$

where

$$\hat{\mathcal{S}}_t = \hat{r}^B_t \tag{B.7}$$

$$\hat{\mathcal{S}}^*_t = \hat{r}^{B*}_t, \tag{B.8}$$

according to (2.26).

Substitute the monetary policy rules into (B.2) – (B.4):

$$\hat{y}_t = \mathbb{E}_t\hat{y}_{t+1} - \frac{1}{\sigma_0}\left(\lambda\phi_\pi\hat{\pi}_{Ht} - \mathbb{E}_t\hat{\pi}_{H,t+1}\right) + K_2\left(\mathbb{E}_t\hat{y}^*_{t+1} - \hat{y}^*_t\right) \tag{B.9}$$

$$\hat{y}^*_t = \mathbb{E}_t\hat{y}^*_{t+1} - \frac{1}{\sigma_0}\left(\lambda\phi_\pi\hat{\pi}^*_{Ft} - \mathbb{E}_t\hat{\pi}^*_{F,t+1}\right) + K_2\left(\mathbb{E}_t\hat{y}_{t+1} - \hat{y}_t\right) \tag{B.10}$$

$$\begin{aligned}&\lambda\phi_\pi\left(\hat{\pi}_{Ht} - \hat{\pi}^*_{Ft}\right) - \mathbb{E}_t\left(\hat{\pi}_{H,t+1} - \hat{\pi}^*_{F,t+1}\right) \\ &= \sigma_0(1 - K_2)\mathbb{E}_t\left[\left(\hat{y}_{t+1} - \hat{y}^*_{t+1}\right) - \left(\hat{y}_t - \hat{y}^*_t\right)\right].\end{aligned} \tag{B.11}$$

Combining the labor-supply conditions (A.8) and (A.9), production functions (A.13) and (A.14), the risk-sharing condition (A.12), and the market-clearing conditions (A.10) and (A.11), the real marginal costs can be derived as

$$\begin{aligned}\widehat{mc}_t &= \phi\hat{n}_t + \sigma\hat{c}_t - \hat{a}_t + (1 - \nu/2)\hat{\tau}_t \\ &= \phi\hat{y}_t - (1 + \phi)\hat{a}_t + \sigma\hat{c}_t + (1 - \nu/2)\hat{\tau}_t \\ &= \phi\hat{y}_t - (1 + \phi)\hat{a}_t + \sigma\hat{y}_t - \sigma(1 - \nu/2)\left(\nu - \frac{\nu - 1}{\sigma}\right)\hat{\tau}_t + (1 - \nu/2)\hat{\tau}_t \\ &= K\hat{y}_t - (1 + \phi)\hat{a}_t + K_1\hat{y}^*_t,\end{aligned} \tag{B.12}$$

where $K = \sigma + \phi - K_1$. The foreign country's counterpart is

$$\widehat{mc}_t^* = K\hat{y}_t^* + K_1\hat{y}_t. \tag{B.13}$$

Combining (B.12) and (B.13) with (A.23) and (A.24), the New Keynesian Phillips curves (NKPCs) are

$$\hat{\pi}_{Ht} = \beta\mathbb{E}_t\hat{\pi}_{H,t+1} + \Theta K\hat{y}_t - \Theta(1+\phi)\hat{a}_t + \Theta K_1\hat{y}_t^* \tag{B.14}$$

$$\hat{\pi}_{Ft}^* = \beta\mathbb{E}_t\hat{\pi}_{F,t+1}^* + \Theta K\hat{y}_t^* + \Theta K_1\hat{y}_t. \tag{B.15}$$

The difference is

$$\hat{\pi}_{Ht} - \hat{\pi}_{Ft}^* = \beta\mathbb{E}_t\left(\hat{\pi}_{H,t+1} - \hat{\pi}_{F,t+1}^*\right) + \Theta(K-K_1) \times \left(\hat{y}_t - \hat{y}_t^*\right) - \Theta(1+\phi)\hat{a}_t. \tag{B.16}$$

Next, we solve the system of equations in (B.11) and (B.16). When $\lambda\phi_\pi > 1$, the Blanchard-Kahn condition is satisfied, and the system has a unique solution, which is (3.1), (3.2). Next, (B.1) implies (3.3).

In our model, $\Theta > 0$, $1 + \phi > 0$, $1 - \rho_a > 0$, $D > 0$, $D + 1 > 0$, $\sigma > 0$, $\sigma_0 > 0$.

- When $\lambda = 1$ and $\phi_\pi > 1$, $\Lambda_a > 0$ and $\lambda\phi_\pi - \rho_a > 0$.
- When $\lambda = 0$, the denominator of Λ_a is a convex quadratic function of ρ_a with one root between 0 and 1 and another root larger than 1. We solve the root within the unit circle

$\overline{\rho}_a = \frac{2\sigma_0(1+\beta)+\Theta(\sigma/D+\phi)(D+1)-\sqrt{[2\sigma_0(1+\beta)+\Theta(\sigma/D+\phi)(D+1)]^2-16\sigma_0^2\beta}}{4\sigma_0\beta}$, and $0 < \rho_a < \overline{\rho}_a$ guarantees $\Lambda_a > 0$. Moreover, $\lambda\phi_\pi - \rho_a < 0$.

B.2. Proof of Corollary 1

When $\sigma = 1$ or $\nu = 2$, $K_1 = K_2 = 0$, so that $\sigma_0 = \sigma$, $K = \sigma + \phi$, and $D = 1$. For the foreign economy, (B.10) and (B.15) yield to

$$\hat{\pi}_{Ft}^* = \hat{y}_t^* = 0. \tag{B.17}$$

The solution to (B.9) and (B.14) for the home economy is

$$\hat{y}_t = \Theta(\lambda\phi_\pi - \rho_a)(1+\phi)\Lambda_a\hat{a}_t \tag{B.18}$$

$$\hat{\pi}_{Ht} = -\Theta(1-\rho_a)(1+\phi)\Lambda_a\hat{a}_t, \tag{B.19}$$

and (B.1) implies $\hat{\tau}_t = \sigma\hat{y}_t$.

B.3. Proof of Proposition 2

When $\lambda = 0$, (3.2) and (3.3) become

$$\hat{y}_t - \hat{y}_t^* = -\rho_a\Theta(1+\phi)(D+1)\Lambda_a\hat{a}_t \tag{B.20}$$

$$\hat{\tau}_t = -\rho_a\Theta(1+\phi)\frac{\sigma(D+1)}{D}\Lambda_a\hat{a}_t. \tag{B.21}$$

First,

$$\frac{\partial D}{\partial \nu} = 2(1-\sigma)(\nu-1) < 0,$$

given $\sigma > 1$. Next, take the derivative of the coefficient in (B.20) with respect to D:

$$\frac{\partial[-\rho_a\Theta(1+\phi)(D+1)\Lambda_a]}{\partial D} = -\rho_a\Theta(1+\phi)\Lambda_a\left\{1 + (D+1)\Lambda_a\left[\rho_a\Theta\left(\phi - \sigma/D^2\right) + \frac{\sigma(1-\beta\rho_a)(1-\rho_a)}{D^2}\right]\right\}. \tag{B.22}$$

Note $\phi - \sigma/D^2$ is an increasing function of D and hence a decreasing function of ν. Therefore, $\phi - \sigma/D^2 \geq \phi - \sigma/D^2|_{\nu=2} = \phi - \sigma > 0$, and $\frac{\partial[-\rho_a\Theta(1+\phi)(D+1)\Lambda_a]}{\partial D} < 0$. That is, $-\rho_a\Theta(1+\phi)(D+1)\Lambda_a$ is increasing in ν and negative. When $\hat{a}_t < 0$, $\hat{y}_t - \hat{y}_t^* \geq \hat{y}_t - \hat{y}_t^*|_{\nu=2} > 0$.

Next, for the coefficient in (B.12),

$$\begin{aligned}
&\frac{\partial\left[-\rho_a\sigma\Theta(1+\phi)\frac{D+1}{D}\Lambda_a\right]}{\partial D} \\
&= -\rho_a\sigma\Theta(1+\phi)\Lambda_a\left\{-\frac{1}{D^2} + (D+1)\Lambda_a/D\left[\rho_a\Theta\left(\phi - \sigma/D^2\right) + \frac{\sigma(1-\beta\rho_a)(1-\rho_a)}{D^2}\right]\right\} \\
&= -\rho_a\sigma\Theta(1+\phi)\Lambda_a\left\{\frac{\sigma(1-\beta\rho_a)(1-\rho_a)(D+1)\Lambda_a/D - 1}{D^2} + (D+1)\Lambda_a\rho_a\Theta\left(\phi - \sigma/D^2\right)/D\right\} \\
&= -\rho_a\sigma\Theta(1+\phi)\Lambda_a\left\{\frac{1}{D^2}\frac{\sigma(1-\beta\rho_a)(1-\rho_a)(D+1)/D - 1/\Lambda_a}{1/\Lambda_a} + (D+1)\Lambda_a\rho_a\Theta\left(\phi - \sigma/D^2\right)/D\right\} \\
&= -\rho_a\sigma\Theta(1+\phi)\Lambda_a\left\{\frac{1}{D^2}\frac{\rho_a\Theta(\sigma/D+\phi)(D+1)}{1/\Lambda_a} + (D+1)\Lambda_a\rho_a\Theta\left(\phi - \sigma/D^2\right)/D\right\}.
\end{aligned} \tag{B.23}$$

D is decreasing in ν: $D \geq D|_{\nu=2} = 1$. Therefore, (B.23) is negative, and $-\rho_a\Theta(1+\phi)\frac{\sigma(D+1)}{D}\Lambda_a$ is negative and increasing in ν, or when $\hat{a}_t < 0$, $\hat{\tau}_t \geq \hat{\tau}_t|_{\nu=2} > 0$.

Appendix C. Setup for quantitative analysis

C.1. Calibration

We calibrate structural parameters according to the standard macro and international literature. The discount factor is $\beta = 0.99$, so the steady-state quarterly risk-free nominal interest rate is 1%. The intertemporal elasticity of substitution is $\sigma = 2$, and the inverse of the Frisch elasticity of labor supply is $\phi = 3$. The elasticity of substitution among differentiated goods is $\theta = 6$, implying the steady-state price markup is 1.2. The price stickiness parameter is $\zeta = 0.75$, meaning the average time between two price adjustments is one year. The persistence of the Taylor rule is $\rho_r = 0.8$, and the sensitivities of the policy rate to inflation and output are $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$. $\nu = 1.5$ implies a significant home bias. The persistence and standard deviation of the TFP shock are $\rho_a = 0.9$ and $\sigma_a = 0.0025$, according to Fernández-Villaverde and Juan (2015). The persistence and standard deviation of the preference shock are $\rho_\xi = 0.9$ and $\sigma_\xi = 0.023$, according to Christiano et al. (2014).

C.2. ELB environment

To create an ELB environment, we impose a series of negative preference shocks on both countries. The shocks occur in periods 1–15, and the total shock size in each country is 23%. These shocks push down the nominal interest rate to zero at period 9 and keep it there until period 20 when there is no unconventional monetary policy.

C.3. Negative TFP shock

In addition to the preference shocks, we hit the home country with a one-time negative TFP shock with a size of -0.5% at period 12.

C.4. Solution method

When $\lambda = 1$, the model is linear, so we use the standard method for solving the rational expectations models. When $\lambda < 1$, we use the occasional binding method of Guerrieri and Iacoviello (2015).

Appendix D. Incomplete asset markets

Following Benigno (2009), there is no longer complete international risk sharing in the model with incomplete asset markets, where only two nominal non-contingent bonds are traded. Then the international risk sharing condition (2.16) no longer holds, and the household's budget constraint (2.2) in the baseline model changes to:

$$P_tC_t - \frac{B_{Ht+1}}{R_t^B} + \frac{\varepsilon_t B_{Ft+1}}{R_t^{B*}} + \frac{\mathscr{l} P_t}{2R_t^{B*}}\left(\frac{\varepsilon_t B_{Ft+1}}{P_t} - \bar{\iota}\right)^2 = \varepsilon_t B_{Ft} - B_{Ht} + W_tN_t + TR_t + \mathcal{D}_t, \tag{D.1}$$

where B_{Ht+1} is the debt issued in units of risk-free nominal bond denominated in H currency, and the nominal interest rate on this bond is R_t^B. B_{Ft+1} is the holding of risk-free nominal bond denominated in units of foreign currency, and the nominal interest rate on this bond is R_t^{B*}. The assumption that households of the home country hold assets denominated in foreign currency and issue debt in the home currency, reflects the net international position of the US economy. We assume a quadratic transition cost when deviating the real foreign bond position from a constant real value, denoted by $\bar{\iota}$; $\mathscr{l}$ is nonnegative, measuring this cost in terms of units of the consumption index, and is rescaled by the factor $1/R_t^{B*}$ for analytical convenience. The quadratic cost serves for the purpose of determining the steady state and getting rid of the indeterminacy problem. TR_t includes government transfer and the revenues obtained from the transaction costs paid by households in the foreign country when trading home country bonds, and $\mathcal{D}_t$ is the profits from firms.

The first-order conditions of home country households with respect to domestic and foreign bonds are:

$$\beta\mathbb{E}_t\left[\frac{\Xi_{t+1}}{\Xi_t}\frac{C_t^\sigma}{C_{t+1}^\sigma}\frac{R_t^B}{\Pi_{t+1}}\right] = 1 \tag{D.2}$$

$$\beta\mathbb{E}_t\left[\frac{\Xi_{t+1}}{\Xi_t}\frac{C_t^\sigma}{C_{t+1}^\sigma}\frac{R_t^{B*}}{\Pi_{t+1}}\frac{\varepsilon_{t+1}}{\varepsilon_t}\right] = 1 + \mathscr{l}\left(\frac{\varepsilon_t B_{Ft+1}}{P_t} - \bar{\iota}\right). \tag{D.3}$$

Utility maximization of households in the foreign country yields the counterparts of Eqs. (D.2) and (D.3). The equilibrium in the asset markets requires that

$$B_{Ht} - B_{Ht}^* = 0 \tag{D.4}$$

$$B_{Ft} - B_{Ft}^* = 0. \tag{D.5}$$

Combining the household's and government budget constraints, we obtain the aggregate budget constraint of the home country:

$$P_tC_t - \frac{B_{Ht+1}}{R_t^B} + \frac{\varepsilon_t B_{Ft+1}}{R_t^{B*}} + \frac{\mathscr{l} P_t}{2R_t^{B*}}\left(\frac{\varepsilon_t B_{Ft+1}}{P_t} - \bar{\iota}\right)^2 = \varepsilon_t B_{Ft} - B_{Ht} + P_{Ht}Y_t + \frac{\mathscr{l}^* P_t}{2R_t^B}\left(\frac{B_{Ht+1}^*}{\varepsilon_t P_t^*} - \bar{\iota}^*\right)^2. \tag{D.6}$$

Appendix E. Data

- Shadow rates are downloaded from Cynthia Wu's website:

https://sites.google.com/site/jingcynthiawu/home/wu-xia-shadow-rates.

- The U.S. macroeconomic variables are downloaded from the Database of the Federal Reserve Bank of St. Louis (FRED) at http://research.stlouisfed.org/fred2/.
  - Real GDP (GDPC): billions of chained 2009 dollars, seasonally adjusted.
  - Real potential GDP (GDPPOT): billions of chained 2009 dollars, not seasonally adjusted.
  - GDP deflator (GDPDEF): index 2009 = 100, seasonally adjusted.
  - Effective federal funds rate (FEDFUNDS): percent.
  - Real output per hour of all persons (nonfarm business sector) (OPHNFB): index 2009 = 100, seasonally adjusted.
  - Hours of all persons (nonfarm business sector) (HOANBS): index 2009 = 100, seasonally adjusted.
  - Civilian noninstitutional population (CNP16OV): thousands of persons.
- The Euro area macroeconomic variables are from the ECB Statistical Data Warehouse at http://sdw.ecb.europa.eu/home.do.
  - Real GDP: reference year 1995, calendar and seasonally adjusted.
  - GDP deflator: index 1995 = 1, calendar and seasonally adjusted.
  - Policy rate: 3-month Euribor.
  - Euro area 19 (fixed composition) total economy labor productivity (per hours worked): index, chain linked volume (rebased), calendar and seasonally adjusted.
  - Euro area 19 (fixed composition) total economy hours worked: hours, calendar and seasonally adjusted.
  - Euro area 19 (fixed composition) employment: thousands of persons, calendar and seasonally adjusted.
- The UK macroeconomic variables are downloaded from the Office for National Statistics at https://www.ons.gov.uk/ and the FRED.
  - Real GDP: seasonally adjusted.
  - GDP deflator: index 1995 = 1, seasonally adjusted.
  - Bank of England policy rate: percent per annum.
  - Output per hour: index 2015 = 100, seasonally adjusted.
  - Average actual weekly hours of work for all workers: millions, seasonally adjusted.
  - Population aged 16 and over: thousands of persons.

Appendix F. VAR for Japan

The pre-ELB and ELB samples for Japan span from 1981Q1 - 1999Q2 and 1999Q3 - 2017Q4. The data include

- Total labor productivity: growth rate same period previous year, seasonally adjusted.
- Hours worked (manufacturing): index 2010 = 100, seasonally adjusted.
- Working age population: aged 15–64, persons, seasonally adjusted.

All series are downloaded from the FRED. Details of the VAR are in Subsection 4.1.

The results are in Fig. F1. We find the impulse response at the ELB is slightly lower than normal times, and the red dashed line is always within the confidence interval in blue. This result suggests that negative supply shocks, such as the Great East Japan earthquake and oil supply shocks, are still contractionary at the ELB in Japan. We conclude the anomaly does not exist.

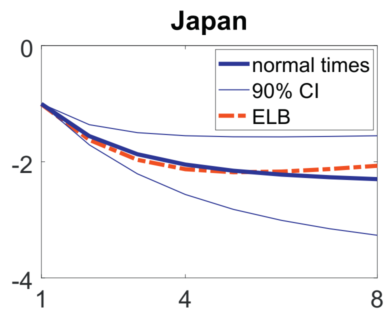

Fig. F1. Impulse response of output to a productivity shock. Notes: Impulse responses of output to a -1% shock to labor productivity growth. The blue solid lines are normal times, with thick lines being the median and thin lines representing 90% confidence intervals. The red dashed line is the median impulse response at the ELB. X-axis: time in quarters; Y-axis: percentage changes in output.

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